Macroscopic quantum phenomena and quantum dissipation

A. O. Caldeira
IFGW

UNICAMP
UNIVERSIDADE ESTADUAL DE CAMPINAS

SP – Brazil

Macroscopic quantum phenomena and quantum dissipation.

Lecture 1:
Elements of superconductivity; superconducting devices and macroscopic quantum phenomena.

Lecture 2:
Quantum dissipation; models and applications.

Lecture 3:
Experimental realizations and superconducting qubits.

Lecture1

A. London theory of superconductivity

Superconductivity is a property common to several metals. Below a given transition temperature they present;

i) transport of charge with no measurable resistance

ii) perfect diamagnetism; Meissner effect

Phenomenologically these two effects can be described by the following equations:

London equations

$$\mathbf{E} = \frac{\partial}{\partial t}(\Lambda \mathbf{J}_s)$$
$$\mathbf{B} = -c\nabla \times (\Lambda \mathbf{J}_s)$$

Combined with the Maxwell equation

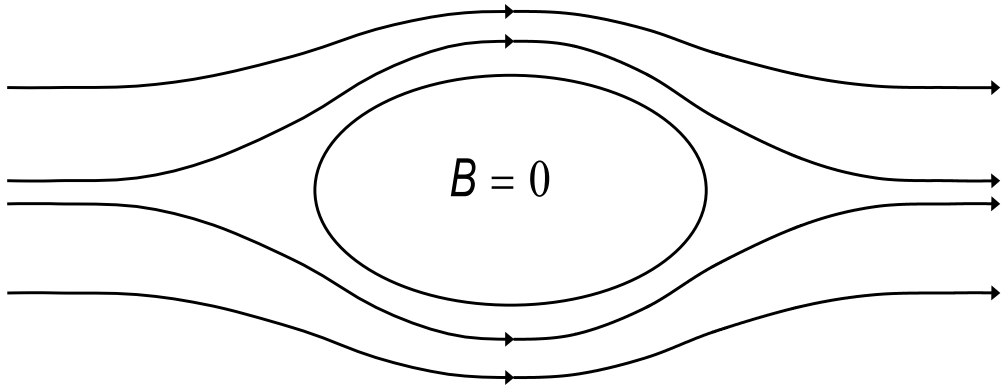
$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}_s$$

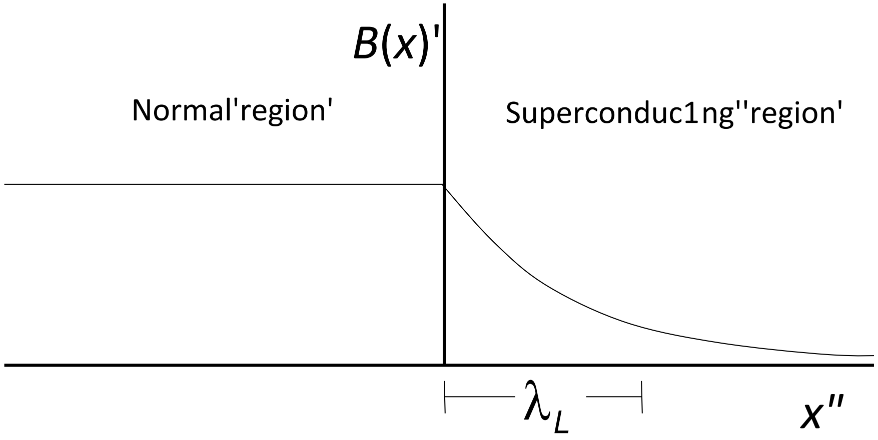
Result in the equation that describes the Meissner effect

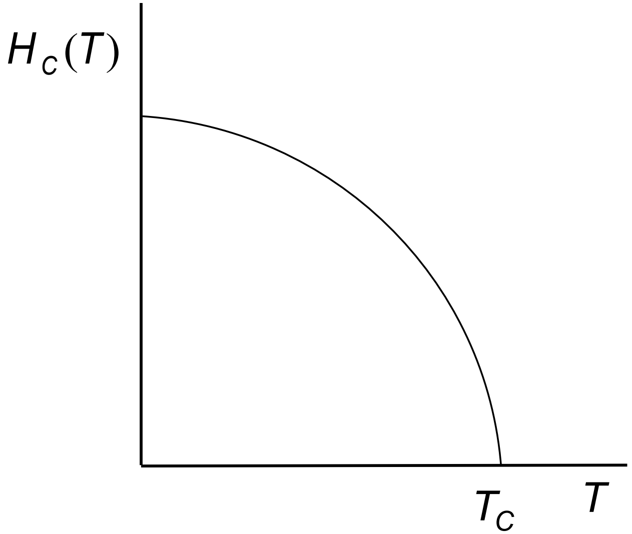
$$\nabla^2 \mathbf{B} = \frac{\mathbf{B}}{\lambda_L^2}$$

where

Meissner effect







If the external scalar potential is zero

$$\mathbf{E} = \frac{\partial}{\partial t}(\Lambda \mathbf{J}_s) = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

This implies that

$$\mathbf{J}_s = -\frac{1}{c\Lambda}(\mathbf{A} - \mathbf{A}_0)$$

Choosing (London gauge)

$$\mathbf{J}_{s\perp} = \frac{1}{c\Lambda}\mathbf{A}_0 = 0$$

London equation in the London gauge

$$\boxed{\mathbf{J}_s = -\frac{1}{c\Lambda}\mathbf{A}}$$

Quantum mechanical argument

Bloch's theorem for the ground state of the superconductor

$$\langle \mathbf{p} \rangle = 0$$

applied to the system in a magnetic field

$$\mathbf{p} = m\dot{\mathbf{r}} + e\frac{\mathbf{A}}{c}$$

yields

$$\mathbf{J}_s = n_s e \langle \mathbf{v}_s \rangle = -\frac{n_s e^2 \mathbf{A}}{mc} = -\frac{1}{c\Lambda}\mathbf{A}$$

and the London penetration depth is given by

$$\lambda_L = \left(\frac{mc^2}{4\pi n_s e^2} \right)^{1/2}$$

Then, for the ground state wavefunction of the superconductor

$$\mathbf{J}_s = \frac{e\hbar}{2mi}\left[\psi_0^*\nabla\psi_0 - \psi_0\nabla\psi_0^*\right] = \frac{e}{m}\ \mathrm{Re}\ [\psi_0^*\mathbf{p}\psi_0] = 0$$

In an external field it changes to $\left(\mathbf{p}\to\mathbf{p}-\frac{e\mathbf{A}}{c}\ \text{and}\ \psi_0\to\psi\right)$

$$\mathbf{J}_s = \frac{e\hbar}{2mi}\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right] - \frac{e^2\mathbf{A}}{mc}\psi^*\psi$$

If the wave function is rigid $\boxed{\psi\approx\psi_0}$

$$\mathbf{J}_s = -\frac{e^2\mathbf{A}}{mc}\psi^*\psi = -\frac{ne^2\mathbf{A}}{mc}$$

“Macroscopic” wave function (order parameter)

Superconducting wave function $\Psi_0(\mathbf{r}_1, ..., \mathbf{r}_k, ..., \mathbf{r}_N)$

Current - carrying wave function (constant velocity)

$$\Psi_{\mathbf{K}}(\mathbf{r}_1, ..., \mathbf{r}_k, ..., \mathbf{r}_N) = \exp\{i \sum_k \mathbf{K} \cdot \mathbf{r}_k\} \Psi_0(\mathbf{r}_1, ..., \mathbf{r}_k, ..., \mathbf{r}_N).$$

Generalization to position dependent velocity

$$\Psi(\mathbf{r}_1, ..., \mathbf{r}_k, ..., \mathbf{r}_N) = \exp\,\{i \sum_k \theta(\mathbf{r}_k)\}\; \Psi_0(\mathbf{r}_1, ..., \mathbf{r}_k, ..., \mathbf{r}_N).$$

Number and current densities

$$n(\mathbf{r}) = \sum_k \int d\mathbf{r}_1 ... d\mathbf{r}_k ... d\mathbf{r}_N \delta(\mathbf{r} - \mathbf{r}_k) \Psi^*(\mathbf{r}_1, ..., \mathbf{r}_k, ..., \mathbf{r}_N) \Psi(\mathbf{r}_1, ..., \mathbf{r}_k, ..., \mathbf{r}_N),$$

$$\mathbf{J}(\mathbf{r}) = \sum_k \int d\mathbf{r}_1 ... d\mathbf{r}_k ... d\mathbf{r}_N \frac{e\hbar}{2mi} \left[\Psi^* \nabla_k \Psi - \Psi \nabla_k \Psi^* \right] \delta(\mathbf{r} - \mathbf{r}_k).$$

These imply a current density with a number density $\Rightarrow$ $$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{m} n(\mathbf{r}) \nabla \theta$$

$$n(\mathbf{r}) = N \int d\mathbf{r}_2 ... d\mathbf{r}_k ... d\mathbf{r}_N \Psi^*(\mathbf{r}, ..., \mathbf{r}_k, ..., \mathbf{r}_N) \Psi(\mathbf{r}, ..., \mathbf{r}_k, ..., \mathbf{r}_N),$$

If $\Psi(\mathbf{r}_1, ..., \mathbf{r}_k, ..., \mathbf{r}_N) = \prod_{k=1}^{N} \psi(\mathbf{r}_k)$

Macroscopic occupation of a single particle state or center-of-mass of a translationally invariant system $\Rightarrow$ $$n(\mathbf{r}) = N \psi^*(\mathbf{r}) \psi(\mathbf{r}),$$

Single particle wavefunction with a new normalization $\Rightarrow$ $$\psi(\mathbf{r}) = \sqrt{n(\mathbf{r})} e^{i\theta(\mathbf{r})}.$$

Non-ideal rigidity $\Rightarrow$ $$\mathbf{J}(\mathbf{r}) = e n_s(\mathbf{r}) \mathbf{v}_s(\mathbf{r}) + e n_N(\mathbf{r}) \mathbf{v}_N(\mathbf{r}),$$

More generally, we can define the 1-particle reduced density operator at zero temperature

$$n_1(\mathbf{r}\,;\mathbf{r}') = N \int d\mathbf{r}_2 ... d\mathbf{r}_k ... d\mathbf{r}_N \Psi_0(\mathbf{r}, ..., \mathbf{r}_k, ..., \mathbf{r}_N)\Psi_0^*(\mathbf{r}', ..., \mathbf{r}_k, ..., \mathbf{r}_N)$$

From which $n(\mathbf{r}) = n_1(\mathbf{r}\,;\mathbf{r})$

Given a general diagonal 1-body operator $\hat{\mathcal{O}}_1 \equiv \sum_i \mathcal{O}_1(\mathbf{r}_i)$ we can write

$$\langle \hat{\mathcal{O}}_1 \rangle = \mathrm{tr}[\hat{n}_1 \hat{\mathcal{O}}_1] = \int d\mathbf{r}\, n_1(\mathbf{r}\,;\mathbf{r})\mathcal{O}_1(\mathbf{r})$$

For a bosonic ground state given by a product state of single particle wave functions $\psi(\mathbf{r})$

$$n(\mathbf{r}) = N\psi^*(\mathbf{r})\psi(\mathbf{r})$$

Defining the 2-particle reduced density operator at zero temperature

$$n_2(\mathbf{x},\mathbf{y};\mathbf{x}',\mathbf{y}') \equiv N(N-1)\int d\mathbf{r}_3 ... d\mathbf{r}_N \Psi_0(\mathbf{x},\mathbf{y},\mathbf{r}_3,...,\mathbf{r}_N)\Psi_0^*(\mathbf{x}',\mathbf{y}',\mathbf{r}_3,...,\mathbf{r}_N)$$

we have, $\langle \hat{\mathcal{O}}_2 \rangle = \frac{1}{2}\mathrm{tr}[\hat{n}_2 \hat{\mathcal{O}}_2] = \frac{1}{2}\int d\mathbf{x}\, d\mathbf{y}\, n_2(\mathbf{x},\mathbf{y}\,;\mathbf{x},\mathbf{y})\mathcal{O}_2(\mathbf{x},\mathbf{y})$

for a general diagonal 2-body operator $\hat{\mathcal{O}}_2 \equiv \frac{1}{2}\sum_{i,j}\mathcal{O}_2(\mathbf{r}_i,\mathbf{r}_j)$

If the ground state is a properly anti-symmetrized product of pairwise particle wave functions

$$n_2(\mathbf{x},\mathbf{y}\,;\mathbf{x},\mathbf{y}) = N(N-1)\,\phi^*(\mathbf{x},\mathbf{y})\phi(\mathbf{x},\mathbf{y})$$

Using the center-of-mass and relative coordinates

$$\mathbf{r} \equiv \frac{1}{2}(\mathbf{x}+\mathbf{y}) \text{ and } \mathbf{u} \equiv \mathbf{x}-\mathbf{y}$$

We have

$$n_1(\mathbf{r}\,;\mathbf{r}') = \frac{1}{N-1}\int d\mathbf{u}\; n_2(\mathbf{r},\mathbf{u}\,;\mathbf{r}',\mathbf{u})$$

Assuming translation invariance $\phi(\mathbf{r},\mathbf{u}) = \psi(\mathbf{r})\chi(\mathbf{u})$

and then $n(\mathbf{r}) \equiv n_1(\mathbf{r}\,;\mathbf{r}) = N\,\psi^*(\mathbf{r})\psi(\mathbf{r})$

as before, but with a new interpretation.

Then $\psi(\mathbf{r}) = \sqrt{n(\mathbf{r})}e^{i\theta(\mathbf{r})}$. But finite temperature effects implies depletion of the condensate (two-fluid model)

⬇

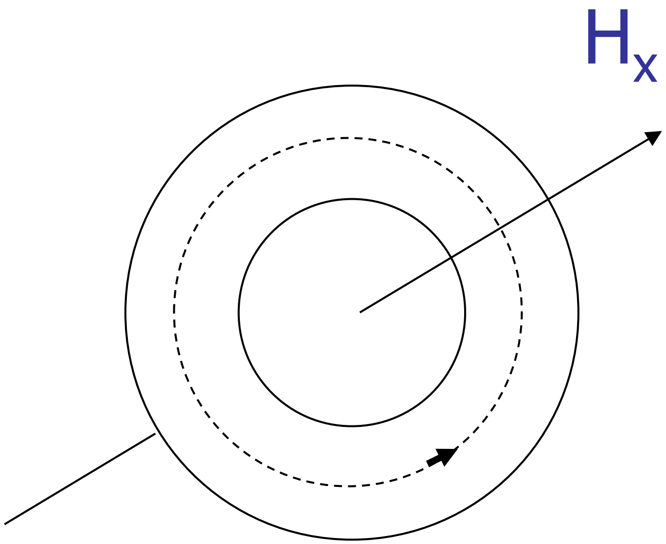
$$\mathbf{J}(\mathbf{r}) = en_s(\mathbf{r})\mathbf{v}_s(\mathbf{r}) + en_N(\mathbf{r})\mathbf{v}_N(\mathbf{r}),$$

and also invalidate the pure state description ➡ density operators, ODLRO, order parameter etc.

C. N. Yang, Rev. Mod. Phys. 34 (4), 694 (1962)

Flux Quantization

Canonical momentum in an external field $\hbar \nabla \theta = m\mathbf{v} + e\mathbf{A}/c$



Integrated along an open path

$$\int_1^2 \left(m\mathbf{v} + \frac{e}{c}\mathbf{A} \right) .d\mathbf{r} = \hbar(\theta_2 - \theta_1)$$

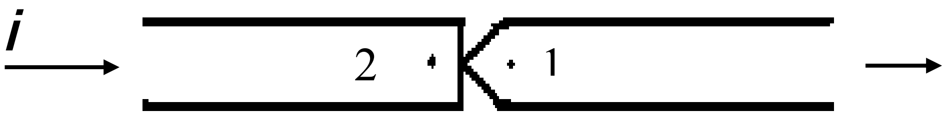
Encircling a hole in a multiply connected region

$$\oint \left(c\Lambda \mathbf{J} + \mathbf{A} \right) \cdot d\mathbf{r} = \frac{2\pi n \hbar c}{2e} \equiv n\phi_0$$

Flux quantization

$$\oint \mathbf{A}.d\mathbf{r} = \int \mathbf{B}.d\mathbf{s} = n\phi_0$$

Josephson Effect



$$\psi_k = \sqrt{n_k}\, e^{i\theta_k}$$

Superconductors coupled by a junction

$$i\hbar\dot{\psi}_1 = E_1\psi_1 + \bar{\Delta}\psi_2$$

$$i\hbar\dot{\psi}_2 = E_2\psi_2 + \bar{\Delta}^*\psi_1$$

Resulting equations for the phase and number density

$$\begin{cases} -\hbar\dot{\theta}_1 = \bar{\Delta}\sqrt{\dfrac{n_2}{n_1}}\,\cos(\theta_2-\theta_1) + E_1 \\ \dfrac{2\bar{\Delta}}{\hbar}\sqrt{n_1 n_2}\,\sin(\theta_2-\theta_1) = -\dot{n}_2 \end{cases}$$

Josephson relations ➡

$$i = i_0\ \sin\Delta\theta$$

$$\Delta\dot{\theta} = \frac{2eV}{\hbar}$$

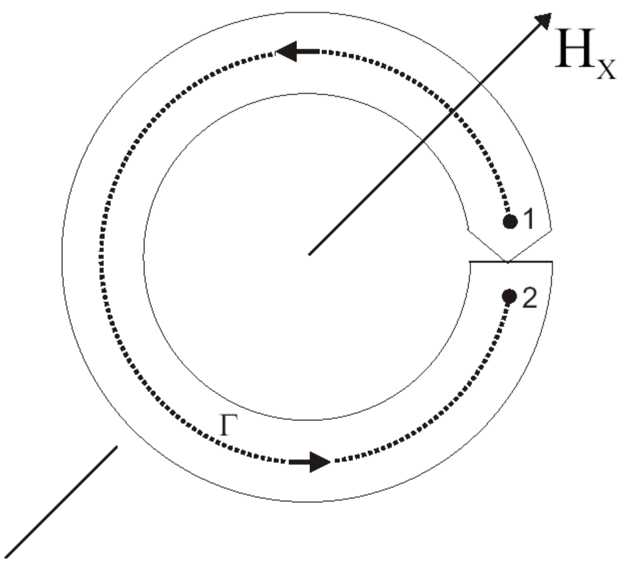
where

$$\Delta\theta \equiv \theta_1 - \theta_2, \quad i_0 \equiv \frac{2n_s\bar{\Delta}}{\hbar} \quad \text{and} \quad \frac{E_2 - E_1}{\hbar} = \frac{2eV}{\hbar}$$

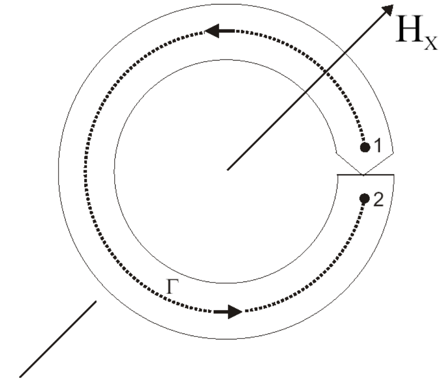
B. Superconducting devices

Superconducting Quantum Interference Devices (SQUIDs)

Superconducting ring closed by a Josephson junction



Modification of the flux quantization rule



$$\int_1^2 \mathbf{J}.d\mathbf{r} = \frac{n_s e\hbar}{m}\int_1^2 \nabla\theta.d\mathbf{r} - \frac{n_s e\hbar}{m}\frac{2\pi}{\phi_0}\int_1^2 \mathbf{A}.d\mathbf{r}.$$

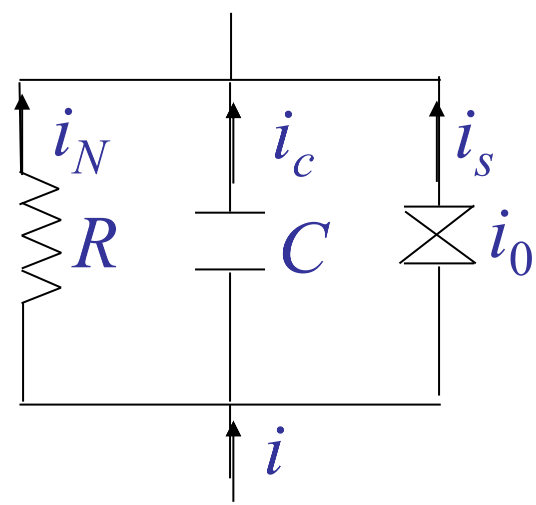
$$\int_\Gamma \nabla\theta.d\mathbf{r} = \oint \nabla\theta.d\mathbf{r} - \int_2^1 \nabla\theta.d\mathbf{r} \quad \Rightarrow \quad \int_\Gamma \nabla\theta.d\mathbf{r} = 2\pi n - \Delta\theta$$

Then $$\phi + \frac{\phi_0}{2\pi}\Delta\theta = n\phi_0.$$ where $$\phi = \phi_x + Li$$

Resistively shunted junction (RSJ) model

$$i_s = i_0 \sin\Delta\theta \qquad i_N = \frac{V}{R} \qquad i_c = C\dot{V}$$

and $$i = i_0 \sin\Delta\theta + \frac{V}{R} + C\dot{V}$$



Using that $V = -\dot{\phi}$ → $$\frac{\phi_x - \phi}{L} = i_0 \sin\frac{2\pi\phi}{\phi_0} + \frac{\dot{\phi}}{R} + C\ddot{\phi}$$

Electromagnetic potential energy

$$U(\phi) = \frac{(\phi - \phi_x)^2}{2L} - \frac{\phi_0 i_0}{2\pi}\cos\frac{2\pi\phi}{\phi_0}$$

Equation of motion for the Total flux in the ring

$$C\ddot{\phi} + \frac{\dot{\phi}}{R} + U'(\phi) = 0$$

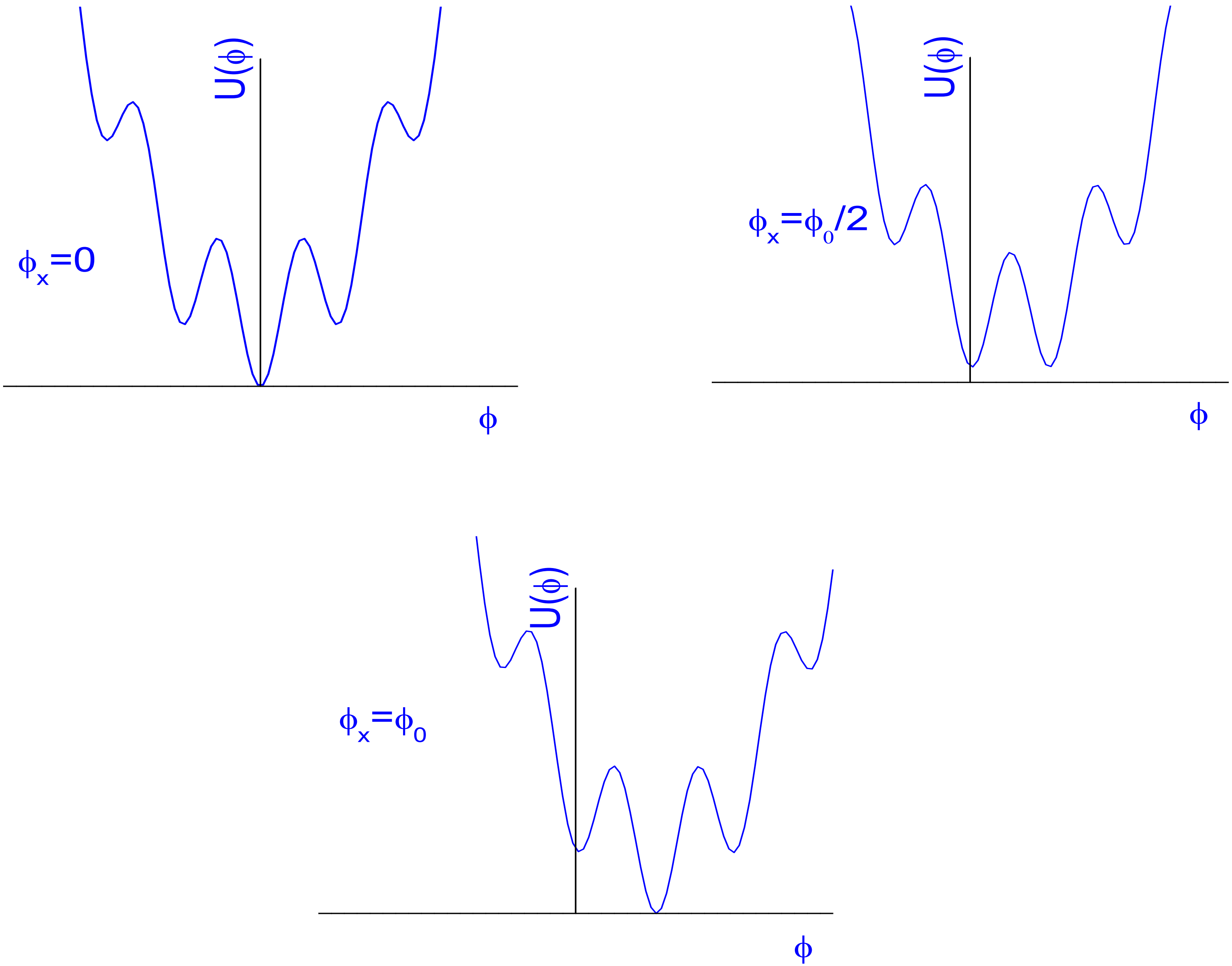
The paradigm

Electromagnetic "potential" energy $$U(\phi) = \frac{(\phi - \phi_x)^2}{2L} - \frac{\phi_0 i_c}{2\pi} \cos \frac{2\pi\phi}{\phi_0}$$

If $\frac{2\pi L i_c}{\phi_0} > 1 \Rightarrow U(\phi)$ has several minima

If $\frac{2\pi L i_c}{\phi_0} \leq 1 \Rightarrow U(\phi)$ has only one minimum

In the first case one has the following possibilities



U(ϕ)
ϕx=0
ϕ
U(ϕ)
ϕx=ϕ0/2
ϕ
U(ϕ)
ϕx=ϕ0
ϕ

The paradigm

In a SQUID with $L \sim 10^{-10}\mathrm{H}$, $C \sim 10^{-12}\mathrm{F}$ and $i_c \sim 10^{-5}\mathrm{A}$

we have $\omega^2 \sim \dfrac{1}{LC} \sim 10^{22}\mathrm{s}^{-2}$. So, for $T = \dfrac{\hbar\omega}{k_B} \lesssim 1\mathrm{K}$

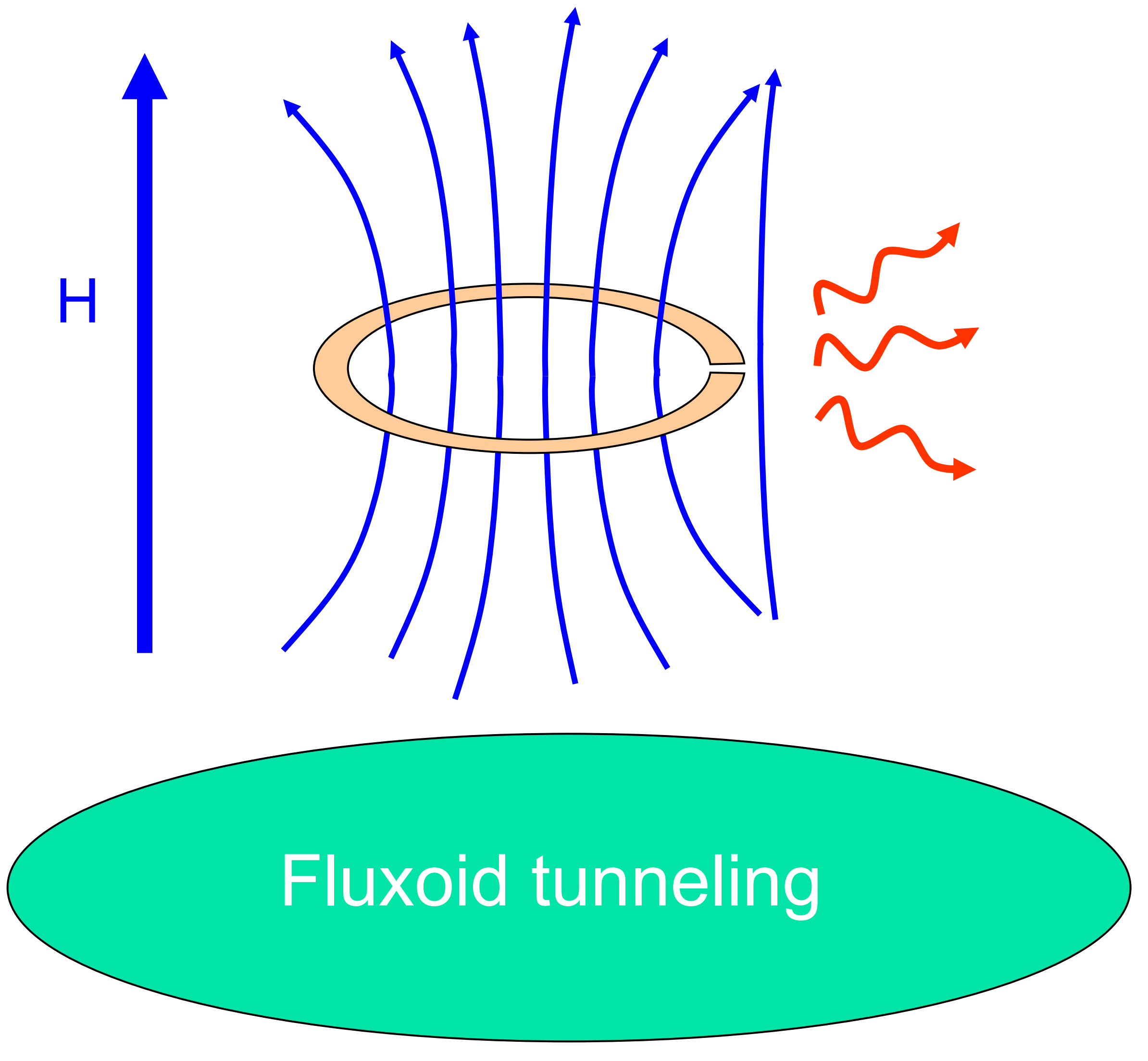
$\Rightarrow$ quantum effects of the electromagnetic field because

$$\phi = \oint \mathbf{A} \cdot d\boldsymbol{\ell}$$

Therefore SQUIDS behave like giant atoms (or molecules):

i) Energy level structure
ii) Decay via quantum tunneling
iii) Coherent tunneling

But…what is tunneling and what is the physics involved in these processes?



Current Biased Josephson Junction (CBJJs)

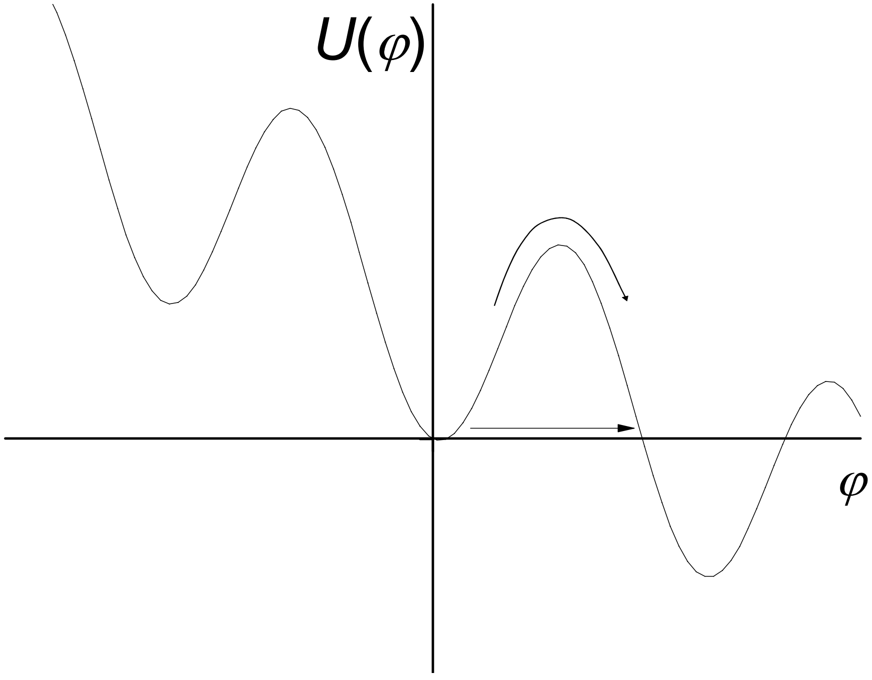
Phase-flux relation $$\phi = -\frac{\phi_0}{2\pi}\,\Delta\theta \equiv \frac{\phi_0}{2\pi}\,\varphi$$

SQUID ring such that $$L \to \infty,\ \phi_x \to \infty \text{ but } \phi_x/L = I_x$$

Washboard potential $$\boxed{U(\varphi) = -I_x\,\varphi - i_0\,\cos\varphi}$$

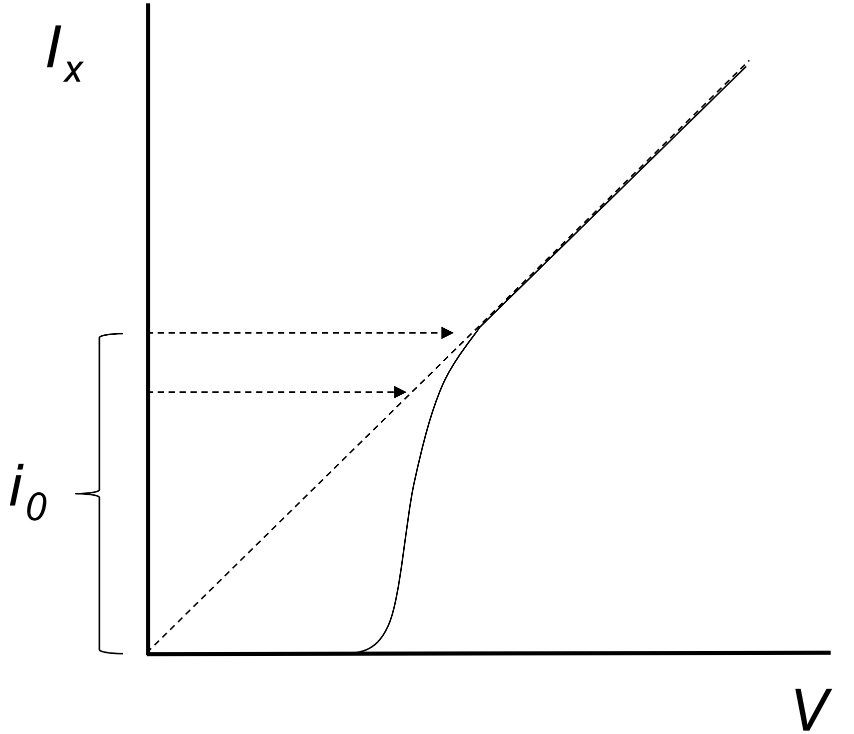
Equation of motion for the phase $$\boxed{\frac{\phi_0}{2\pi}\,C\,\ddot{\varphi} + \frac{\phi_0}{2\pi R}\,\dot{\varphi} + U'(\varphi) = 0}$$

Josephson coupling energy $$\boxed{E_J \equiv \frac{\phi_0 i_0}{2\pi}}$$

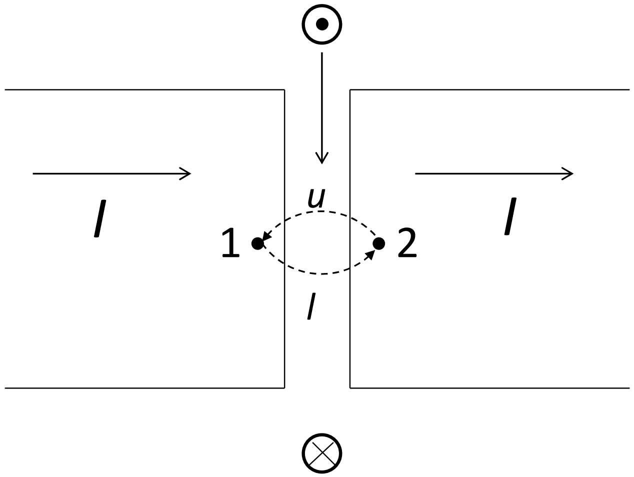


Washboard potential

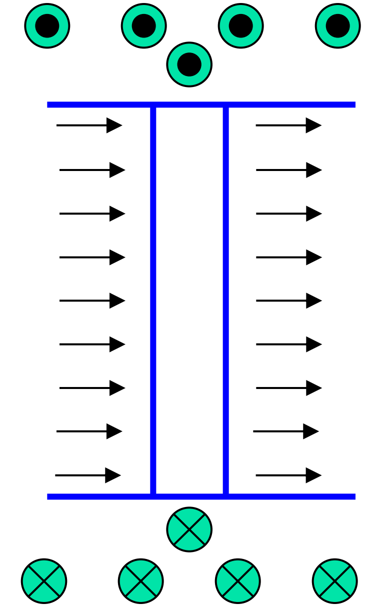
V x I characteristic of the CBJJ



Phase slip



Quantum phase slips



If one vortex crosses the junction:

$$\oint \nabla\theta \cdot d\mathbf{l} = \int_1^2 (\nabla\theta)_l \cdot d\mathbf{l} + \int_2^1 (\nabla\theta)_u \cdot d\mathbf{l} = 2\pi$$

$$(\theta_1 - \theta_2)_u - (\theta_1 - \theta_2)_l \equiv \Delta\theta_u - \Delta\theta_l = 2\pi$$

If N vortices cross the junction:

$$\frac{d\Delta\theta}{dt} = 2\pi \frac{dN}{dt} \quad \Rightarrow \quad V = \phi_0 \frac{dN}{dt} \quad \Rightarrow \quad V = \phi_0 n_v v_L d$$

Cooper Pair Boxes (CPBs)

Charging energy $E_C = \dfrac{e^2}{2C}$.   If $E_C \gg E_J$

“ Nearly free-electron model ” for the phase in a periodic potential

$$H_0 = \frac{Q^2}{2C} + U(\varphi) \qquad \text{where} \qquad Q = -i\hbar \frac{\mathrm{d}}{\mathrm{d}(\phi_0 \varphi / 2\pi)}$$

Bloch´s theorem $\psi_{n\,q}(\varphi) = \exp\left\{ i \left(\dfrac{q}{2e} \right) \varphi \right\} \; u_n(\varphi)$

with $\quad u_n(\varphi + 2\pi) = u_n(\varphi)$

where $\quad q(t) = q_0 + Q_x(t) \quad$ and $\quad Q_x(t) = \int_{t_0}^{t} dt' I_x(t')$

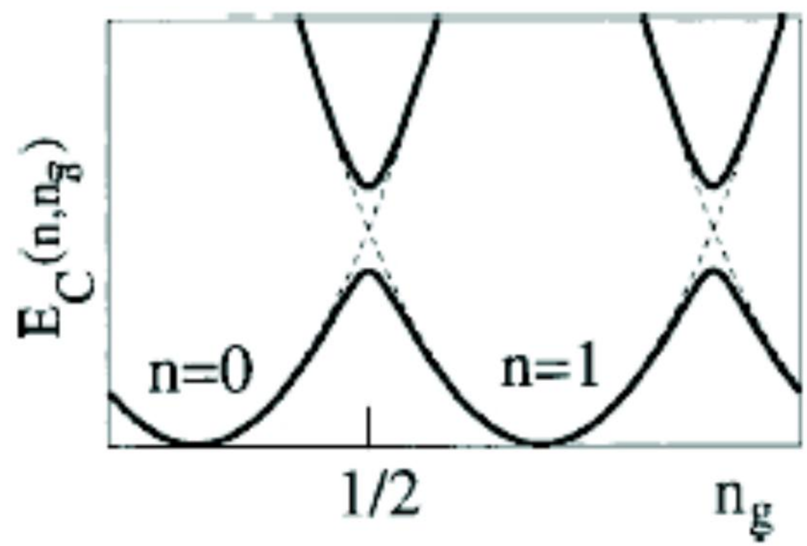
New Schrödinger equation (adiabatic approximation)

$$\mathcal{H}_q u_n(\varphi) = \frac{(Q+q)^2}{2C} u_n(\varphi) + U(\varphi) u_n(\varphi) \;=\; E_n(q) u_n(\varphi)$$

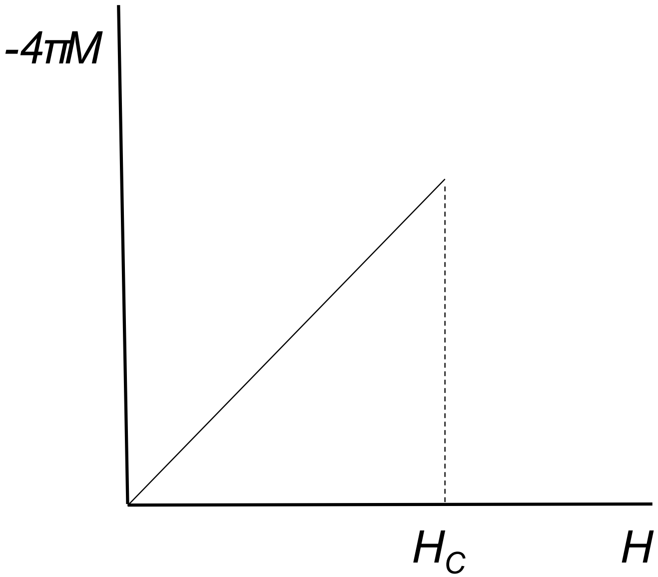
where $\quad Q = -2ie\dfrac{\partial}{\partial\varphi}$

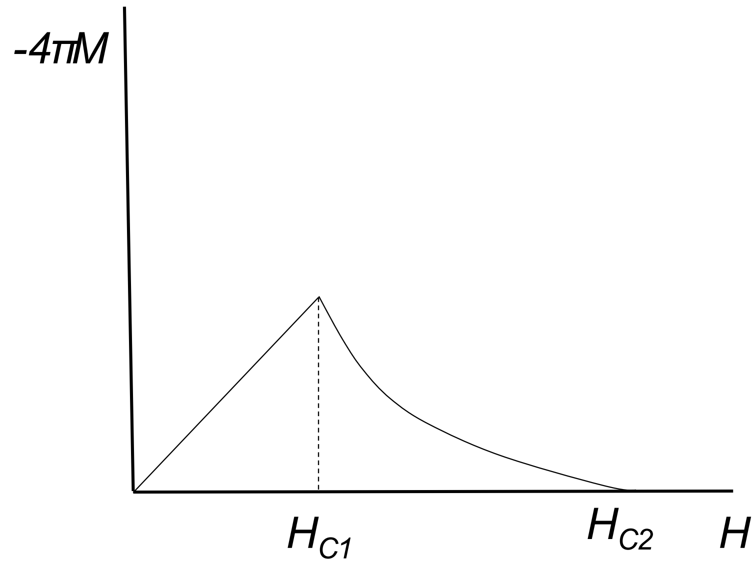
with $\quad u_n(\varphi + 2\pi) = u_n(\varphi)$

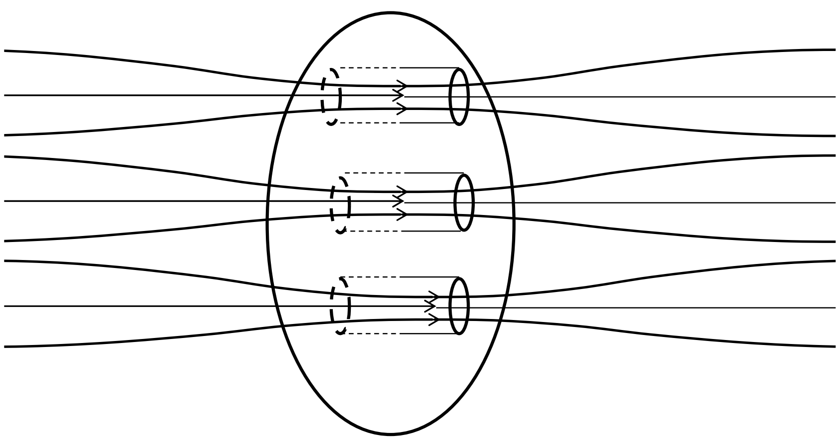
Band structure of the CPB

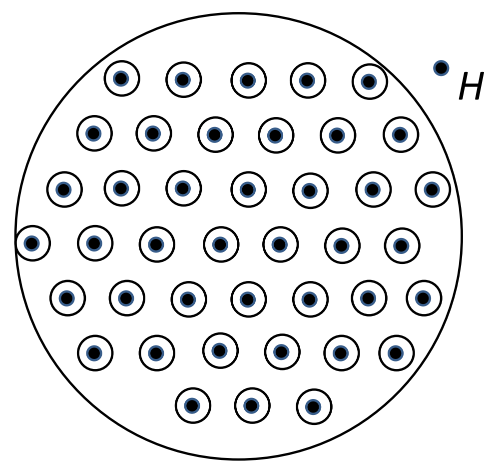


C. Vortices in superconductors









New characteristic length, *the coherence length* $\xi = \xi(T)$

$\xi_0 = \xi(0)$ is basically the radius of the Cooper pair

Estimate of this radius:

Energy cost to create an excitation in a metal is zero, but in a superconductor

$$E_F - \Delta < \frac{p^2}{2m} < E_F + \Delta$$

$\Delta << E_F$ ➡ $\delta p = 2\Delta / v_F$

Uncertainty principle ➡ $\delta x \propto \dfrac{\hbar}{\delta p} = \dfrac{\hbar v_F}{2\Delta}$ $\xi_0 = \dfrac{\hbar v_F}{\pi \Delta}$

Temperature dependence is the same for $\xi(T)$ and $\lambda(T)$

What matters is λ_L/ξ_0 . From the Ginzburg-Landau theory

Pure metals $\lambda_L/\xi_0 < 1/\sqrt{2}$ ➡ Type I superconductors
Pippard theory

Alloys $\lambda_L/\xi_0 > 1/\sqrt{2}$ ➡ Type II superconductors
London theory

The supercurrent $\mathbf{J_s}(\mathbf{r})$ is obtained from an average of $\mathbf{A}(\mathbf{r}')$ over a region such that $|\mathbf{r}-\mathbf{r}'| < \xi_0$ in the Pippard theory

Condensation energy

Order parameter and penetration depth change abruptly at a surface

$$F_N - F_S = \frac{H_c^2}{8\pi}$$

If not

$$F_N - F_S = \frac{H_c^2}{8\pi} + \frac{H_c^2}{8\pi}\frac{(\lambda - \xi)S}{V}$$

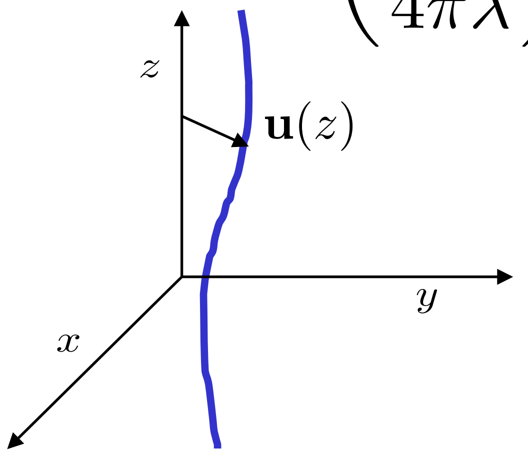
In a cylinder the EM field penetrates the sample in tubes: *vortices*. Creation of as many vortices as possible to reduce the superconducting free energy. It is halted by *vortex-vortex interaction*.

Vortices

Energy per unit length of a vortex in a type II superconductor

$$\epsilon_l = \frac{1}{8\pi} \int_{r>\xi} dS(\,\lambda^2\,|\nabla \times \mathbf{h}(\mathbf{r})|^2 + h^2(\mathbf{r}))$$

For $\kappa \equiv \lambda_L/\xi_0 >> 1\,,\quad \epsilon_l = \epsilon_0 \ln \kappa$ with $\epsilon_0 = \left(\frac{\phi_0}{4\pi\lambda}\right)^2$



For a distorted vortex tube with
$\mathbf{u}(z) = [u_x(z), u_y(z)]$

$$\mathcal{F}_{\rm el} = \int dz\,\epsilon_l \left\{\left[1 + \left(\frac{\partial \mathbf{u}}{\partial z}\right)^2\right]^{1/2} - 1\right\} \approx \int dz\,\frac{\epsilon_l}{2}\left(\frac{\partial \mathbf{u}}{\partial z}\right)^2$$

Vortices

Linear density of mass of the vortex line is $m_l = \frac{2}{\pi^3} m_e k_F$

Force on a vortex: in general *Lorentz force:* $\mathbf{f}_L(\mathbf{r}) = \mathbf{J}_s(\mathbf{r}) \times \frac{\mathbf{\Phi}_0}{c}$

Magnus force due to the motion relative to the superfluid velocity

$$\mathbf{f}_M(\mathbf{r}) = \rho_s[\mathbf{v}_s(\mathbf{r}) - \mathbf{v}_l] \times \frac{\mathbf{\Phi}_0}{c}$$

Dissipative and Hall effects on a stiff line $m_l \dot{\mathbf{v}}_l + \eta_l \mathbf{v}_l + \alpha_l \mathbf{v}_l \times \hat{\mathbf{z}} = \mathbf{f}_L$

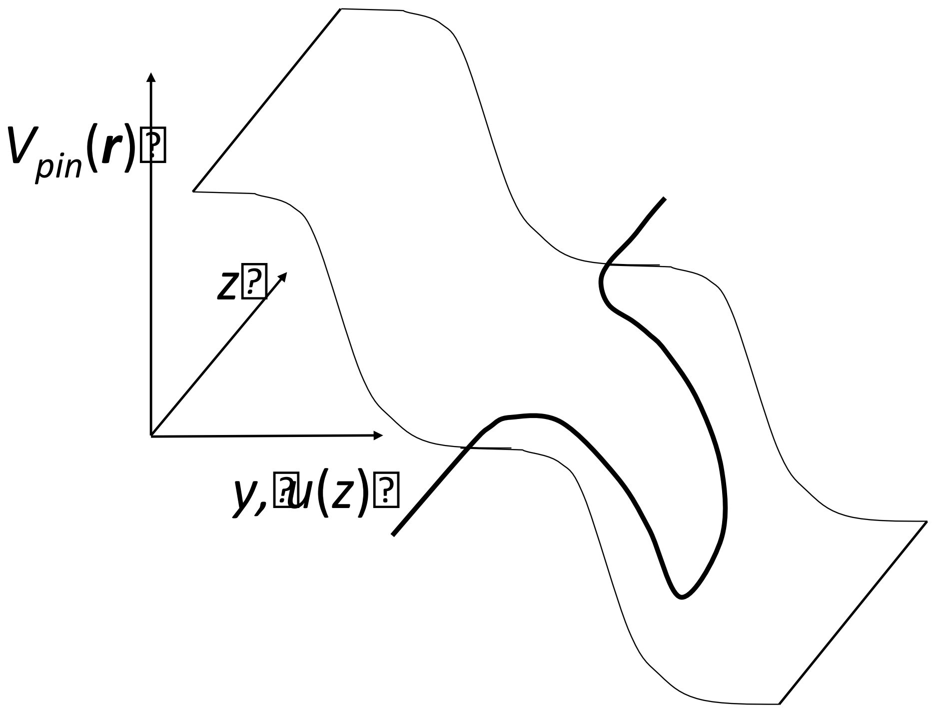
$$\begin{cases} \eta_l = \dfrac{\phi_0}{c} \rho_s \dfrac{\omega_0 \tau_r}{1 + \omega_0^2 \tau_r^2} \\ \alpha_l = \dfrac{\phi_0}{c} \rho_s \dfrac{\omega_0^2 \tau_r^2}{1 + \omega_0^2 \tau_r^2} \end{cases}$$

Elastic line

$$m_l \frac{\partial^2 \mathbf{u}(z,t)}{\partial t^2} + \eta_l \frac{\partial \mathbf{u}(z,t)}{\partial t} + \alpha_l \frac{\partial \mathbf{u}(z,t)}{\partial t} \times \hat{\mathbf{z}} - \epsilon_l \frac{\partial^2 \mathbf{u}(z,t)}{\partial z^2} + \frac{\partial V_{pin}(\mathbf{u}(z,t))}{\partial \mathbf{u}(z,t)} = \mathbf{f}_L$$

We have the potential energy functional:

$$\mathcal{H}[\mathbf{u}(z,t)] = \int_{-\infty}^{+\infty} dz \left[\frac{\epsilon_l}{2} \left(\frac{\partial \mathbf{u}(z,t)}{\partial z} \right)^2 + V_{pin}(z, \mathbf{u}(z,t)) - \mathbf{f}_L \cdot \mathbf{u}(z,t) \right]$$



D. Macroscopic Quantum Phenomena

Metastable configuration of the SQUID corresponds to a state of the condensate that carries zero current

$$|\Phi_i\rangle = |A_i\rangle\ |\psi_i\rangle$$

Stable configuration carries a finite current

$$|\Phi_f\rangle = |A_f\rangle\ |\psi_f\rangle$$

Total decaying state (caution)

$$|\Phi_D(t)\rangle \approx e^{-\frac{\gamma t}{2}}\ |A_i\rangle\ |\psi_i\rangle + \sqrt{(1-e^{-\gamma t})}\,|A_f\rangle\ |\psi_f\rangle$$

Another possibility is a bistable coherent oscillation between states carrying different currents

$$|\Phi_B(t)\rangle\ \approx\ a(t)\,|A_i\rangle\,|\psi_i\rangle + b(t)\,|A_f\rangle\,|\psi_f\rangle$$

They are both Schrödinger - cat like states

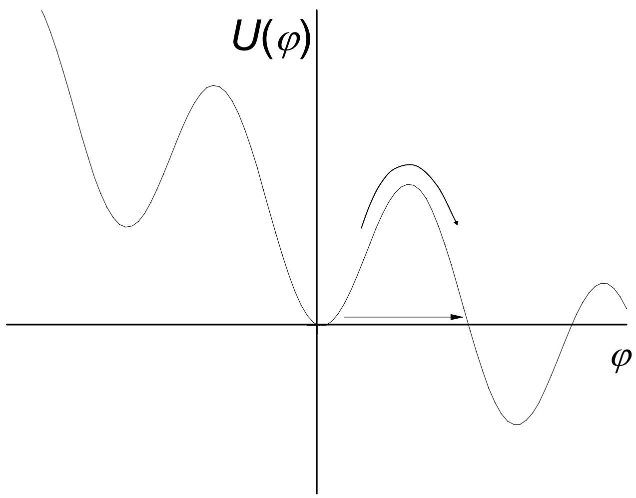
$$\Phi_B(\mathbf{r},\mathbf{r}_1,...,\mathbf{r}_N,t) = a(t)\,A_i(\mathbf{r})\,\psi_i(\mathbf{r},\mathbf{r}_1,...,\mathbf{r}_N) + b(t)\,A_f(\mathbf{r})\,\psi_f(\mathbf{r},\mathbf{r}_1,...,\mathbf{r}_N)$$

that differ from either a macroscopically occupied single particle state (the condensate wavefunction) or a Josephson Effect – like wavefunction

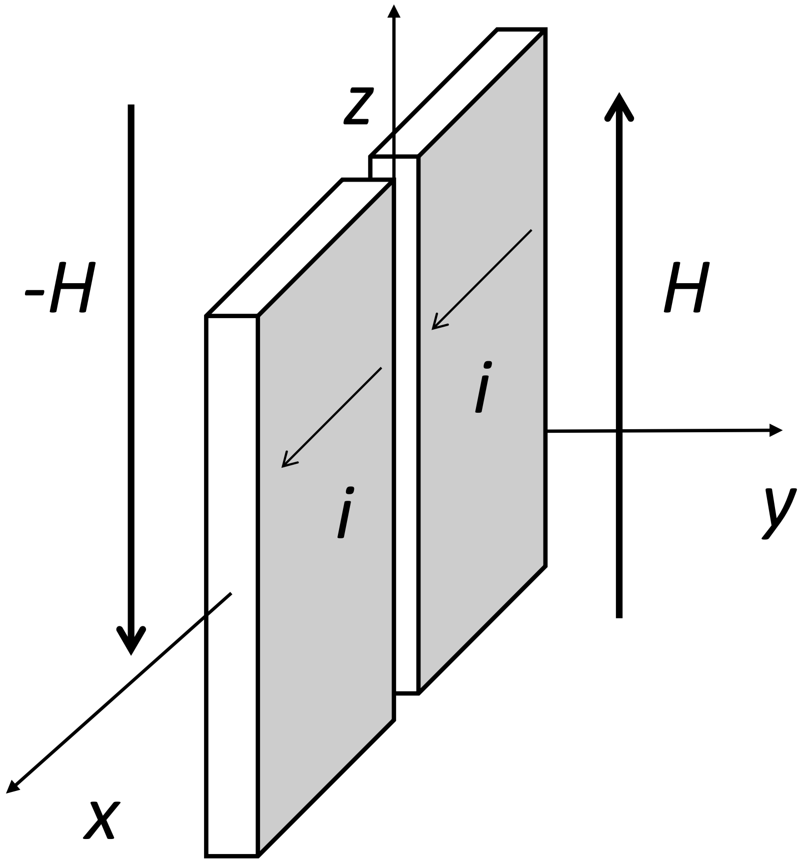
$$\varphi(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2,...,\mathbf{x}_{N/2},\mathbf{y}_{N/2}) = \prod_{i=1}^{N/2}\left[a_R^{(i)}\,\varphi_R(\mathbf{x}_i,\mathbf{y}_i) + a_L^{(i)}\,\varphi_L(\mathbf{x}_i,\mathbf{y}_i)\right]$$

where $\varphi(\mathbf{x}_i,\mathbf{y}_i) = a_R^{(i)}\,\varphi_R(\mathbf{x}_i,\mathbf{y}_i) + a_L^{(i)}\,\varphi_L(\mathbf{x}_i,\mathbf{y}_i)$

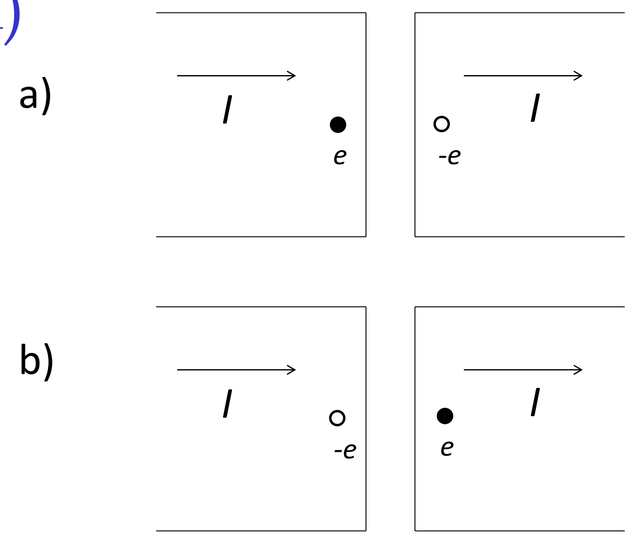
Phase slip (phase representation)



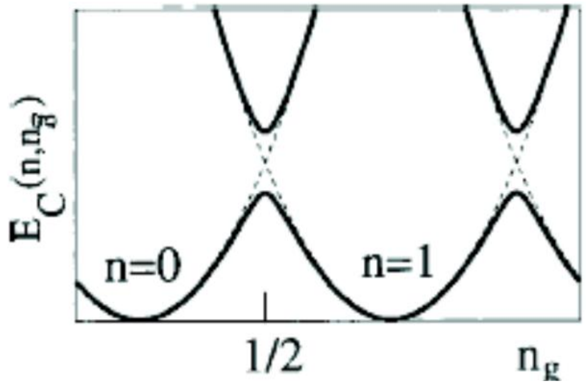
$$\phi_J = (a\,\phi_1 + b\,\phi_2)^{N/2}$$



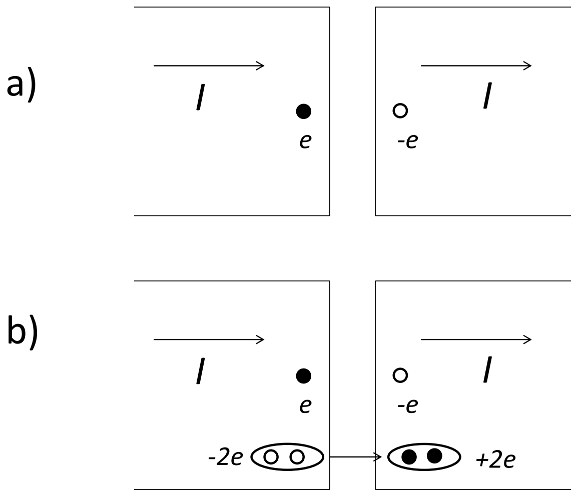
Phase slip (charge representation)



$$|0\rangle = |e,0\rangle_L \otimes |-e,0\rangle_R$$



$$|1\rangle = |e,-2e\rangle_L \otimes |-e,2e\rangle_R$$



$$|\pm\rangle = |0\rangle \pm |1\rangle$$

Lecture 2

A. Dissipative quantum systems

Brownian particle; a paradigm

$$M\ddot{q} + \eta\dot{q} + V'(q) = f(t)$$ where

$$\langle f(t) \rangle = 0$$ and

$$\langle f(t) f(t') \rangle = 2\eta k_B T \delta(t - t')$$

What about quantum mechanics?

System + reservoir approach

Dissipative systems are such that

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_I + \mathcal{H}_R$$

Averages of interest

Given $\hat{\rho}(t) \equiv \sum_{\Psi} p_{\Psi} |\Psi(t)\rangle\langle\Psi(t)| = e^{-i\mathcal{H}t/\hbar}\hat{\rho}(0)e^{i\mathcal{H}t/\hbar}$

$$\langle\hat{O}(q,p)\rangle = \mathrm{tr}_{RS}\{\hat{\rho}(t)\hat{O}\} = \mathrm{tr}_S\{[\mathrm{tr}_R\hat{\rho}(t)]\hat{O}\} = \mathrm{tr}_S\{\tilde{\rho}(t)\hat{O}\}$$

$\tilde{\rho}(t) \equiv \mathrm{tr}_R\hat{\rho}(t)$ is the reduced density operator of the system

Quantum dynamics (general approach)

Time evolution of the density operator of the universe (S + R)

$$\hat{\rho}(x,\mathbf{R},y,\mathbf{Q},t) =$$
$$\int\int\int\int dx'dy'd\mathbf{R}'d\mathbf{Q}'\,K(x,\mathbf{R},t;x',\mathbf{R}',0)\,K^*(y,\mathbf{Q},t;y',\mathbf{Q}',0)\,\hat{\rho}(x',\mathbf{R}',y',\mathbf{Q}',0)$$

where $\mathbf{R} = (R_1,...,R_N)$ and

$$K(x,\mathbf{R},t;x',\mathbf{R}',0) = \left\langle x,\mathbf{R}\left|e^{-i\mathcal{H}t/\hbar}\right|x',\mathbf{R}'\right\rangle = \int_{x'}^{x}\int_{\mathbf{R}'}^{\mathbf{R}}\mathcal{D}x(t')\,\mathcal{D}\mathbf{R}(t')\,\exp\left\{\frac{i}{\hbar}S[x(t'),\mathbf{R}(t')]\right\}$$

Initial condition $\hat{\rho}(x',\mathbf{R}',y',\mathbf{Q}',0) = \hat{\rho}_S(x',y',0)\,\hat{\rho}_R(\mathbf{R}',\mathbf{Q}',0)$

leads us to $\tilde{\rho}(x,y,t) = \int\int dx'dy'\,\mathcal{J}(x,y,t;x',y',0)\,\tilde{\rho}(x',y',0)$ where

$$\mathcal{J}(x,y,t;x',y',0) = \int\int\int d\mathbf{R}'d\mathbf{Q}'d\mathbf{R}\left\{K(x,\mathbf{R},t;x',\mathbf{R}',0)\,K^*(y,\mathbf{R},t;y',\mathbf{Q}',0)\tilde{\rho}_R(\mathbf{R}',\mathbf{Q}',0)\right\}$$

Feynman – Vernon representation of the time evolution of the reduced density operator (separable initial condition)

$$\mathcal{J}(x,y,t;x',y',0)=\int_{x'}^{x}\int_{y'}^{y}\mathcal{D}x(t')\,\mathcal{D}y(t')\,\exp\left\{\frac{i}{\hbar}S_0[x(t')]\right\}\,\exp\left\{-\frac{i}{\hbar}S_0[y(t')]\right\}\mathcal{F}[x(t'),y(t')]$$

Influence functional

$$\mathcal{F}[x(t'),y(t')]=\int\int\int d\mathbf{R}'\,d\mathbf{Q}'\,d\mathbf{R}\,\rho_R(\mathbf{R}',\mathbf{Q}',0)\int_{\mathbf{R}'}^{\mathbf{R}}\int_{\mathbf{Q}'}^{\mathbf{R}}\mathcal{D}\mathbf{R}(t')\,\mathcal{D}\mathbf{R}(t')\,\times$$

$$\times\,\exp\frac{i}{\hbar}\Big\{S_I[x(t'),\mathbf{R}(t')]+S_R[\mathbf{R}(t')]\Big\}\,\times\,\exp-\frac{i}{\hbar}\Big\{S_I[y(t'),\mathbf{Q}(t')]+S_R[\mathbf{Q}(t')]\Big\}$$

where $S=S_0[q]+S_I[q,q_k]+S_R[q_k]$

B. Phenomenological approach: the minimal model

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_I + \mathcal{H}_R$$

Two choices:

i) Realistic model for $\mathcal{H}_S$

i) Simple model for $\mathcal{H}_S +$ constraint (Langevin equation in the classical limit)

Choosing the second one

$$L_S = \frac{1}{2} M \dot{q}^2 - V(q) \qquad L_I = \sum_k C_k \, q_k \, q$$

$$L_R = \sum_k \frac{1}{2} m_k \, \dot{q}_k^2 - \sum_k \frac{1}{2} m_k \, \omega_k^2 \, q_k^2 \qquad L_{CT} = -\sum_k \frac{1}{2} \frac{C_k^2}{m_k \, \omega_k^2} q^2$$

Equations of motion

$$M\ddot{q} = -V'(q) + \sum_k C_k\, q_k - \sum_k \frac{C_k^2}{m_k\,\omega_k^2}\, q$$

$$m_k\,\ddot{q}_k = -m_k\,\omega_k^2\, q_k + C_k\, q$$

Laplace transform of the k^th^ coordinate of the bath

$$\tilde{q}_k(s) = \frac{\dot{q}_k(0)}{s^2+\omega_k^2} + \frac{s\,q_k(0)}{s^2+\omega_k^2} + \frac{C_k\,\tilde{q}(s)}{m_k\ (s^2+\omega_k^2)}$$

Equation of motion of the variable of interest as a function of time

$$M\ddot{q} + V'(q) + \sum_k \frac{C_k^2}{m_k\,\omega_k^2}\,\frac{1}{2\,\pi\,i}\int_{\varepsilon-i\,\infty}^{\varepsilon+i\,\infty}\frac{s^2\,\tilde{q}(s)}{s^2+\omega_k^2}\,e^{s\,t}\,ds$$

$$= -\frac{1}{2\,\pi\,i}\int_{\varepsilon-i\,\infty}^{\varepsilon+i\,\infty}\sum_k C_k\left\{\frac{\dot{q}_k(0)}{s^2+\omega_k^2} + \frac{s\,q_k(0)}{s^2+\omega_k^2}\right\}e^{s\,t}\,ds,$$

The last term on the LHS is $\frac{d}{dt}\left\{\sum_k \frac{C_k^2}{m_k\,\omega_k^2}\,\frac{1}{2\,\pi\,i}\int_{\varepsilon-i\,\infty}^{\varepsilon+i\,\infty}\frac{s\,\tilde{q}(s)}{s^2+\omega_k^2}\,e^{s\,t}\,ds\right\}$

or $$\frac{d}{dt}\left\{\sum_k \frac{C_k^2}{m_k\,\omega_k^2}\int_0^t \cos\left[\omega_k\,\left(t-t'\right)\right]\,q(t')\,dt'\right\}.$$

The term on the RHS is a fluctuating force such that

Equipartition theorem

$$\begin{cases} \langle q_k(0)\rangle = \bar{q}_k \quad \text{and} \quad \langle \dot{q}_k(0)\rangle = \langle \dot{q}_k(0)\,\Delta q_k(0)\rangle = 0, \\ \langle \dot{q}_k(0)\,\dot{q}_{k'}(0)\rangle = \dfrac{k_B\,T}{m_k}\,\delta_{kk'}, \\ \langle \Delta q_k(0)\,\Delta q_{k'}(0)\rangle = \dfrac{k_B\,T}{m_k\,\omega_k^2}\,\delta_{kk'} \end{cases}$$

Transforming sums into integrals through the spectral function $J(\omega) = \frac{\pi}{2} \sum_k \frac{C_k^2}{m_k \, \omega_k} \, \delta\left(\omega - \omega_k\right)$

and modelling it as

$$J(\omega) = \begin{cases} A_s \omega^s & \text{if} \quad \omega < \Omega \\ 0 & \text{if} \quad \omega > \Omega, \end{cases} \qquad \text{where} \;\; [A_s] = M T^{s-2}$$

one shows that for *ohmic dissipation,* $s = 1$ and $[A_s] = \eta$, it yields

$$M \, \ddot{q} + \eta \, \dot{q} + V'(q) = f(t)$$

$$\langle f(t) \rangle = 0,$$

$$\langle f(t) \, f(t') \rangle = 2 \, \eta \, k_B \, T \, \delta\left(t - t'\right)$$

Classical Langevin equation for the Brownian motion

0 < *s* < 1, *subohmic* case and *s* >1 , *superohmic* case.

Other forms of the same model: *velocity coupling*

If we write the Lagrangian of the whole system as

$$L = L_S + L_R + \tilde{L}_I$$ (notice there is no counter – term!)

where $\tilde{L}_I = \sum_k \tilde{C}_k \, q \, \dot{q}_k$ with $C_k \equiv \tilde{C}_k \, \omega_k$

and go over to the Hamiltonian formalism, we recover the original model (with the appropriate counter – term) after performing the canonical transformation

$$p \to p, \; q \to q, \; p_k \to m_k \, \omega_k \, q_k \,, \; \text{and} \; q_k \to \frac{p_k}{m_k \, \omega_k}$$

Other forms of the same model: *translation invariant bath*

If in the original system we make the replacement

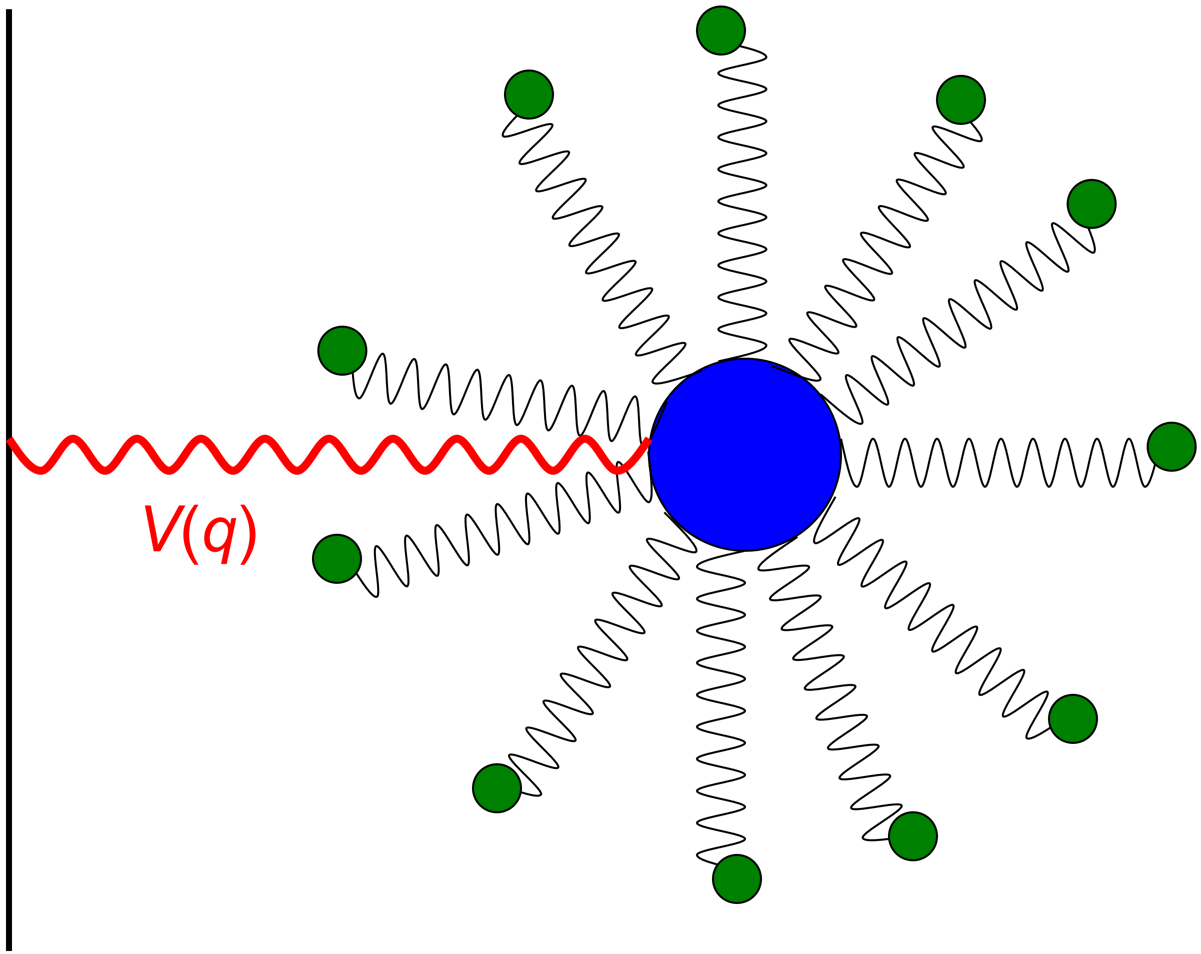
$$q_k \to \frac{C_k q_k}{m_k\,\omega_k^2} \quad \text{and} \quad p_k \to \frac{m_k\,\omega_k^2\,p_k}{C_k}$$

we get $\mathcal{H} = \frac{p^2}{2\,M} + V(q) + \sum_k \left\{ \frac{p_k^2}{2\,\mu_k} + \frac{1}{2}\mu_k\,\omega_k^2\,(q_k - q)^2 \right\}$

where $\mu_k \equiv \frac{C_k^2}{m_k \omega_k^4}$ ⟹ $J(\omega) = \frac{\pi}{2} \sum_k \mu_k\,\omega_k^3\,\delta(\omega - \omega_k)$

Manifestly translation invariant if $V(q) = 0$!

Mechanical analogue



Manifestly translation invariant if $V(q) = 0$!

The influence functional

$$\mathcal{F}[x(t'),y(t')] = \int\int\int d\mathbf{R}'\,d\mathbf{Q}'\,d\mathbf{R}\,\rho_R(\mathbf{R}',\mathbf{Q}',0)\int_{\mathbf{R}'}^{\mathbf{R}}\int_{\mathbf{Q}'}^{\mathbf{R}}\mathcal{D}\mathbf{R}(t')\,\mathcal{D}\mathbf{R}(t')\ \times$$
$$\times\ \exp\left\{\frac{i}{\hbar}\Big[S_I[x(t'),\mathbf{R}(t')]-S_I[y(t'),\mathbf{Q}(t')]+S_R[\mathbf{R}(t')]-S_R[\mathbf{Q}(t')]\Big]\right\},$$

Forced harmonic oscillator action

$$\begin{aligned}
S_{cl}^{(k)} \;=\;& \frac{m_k\omega_k}{2\sin\omega_k t}\Big[\Big(R_k^2+{R_k'}^2\Big)\cos\omega_k t - 2R_kR_k' \\
&+\frac{2C_kR_k}{m_k\omega_k}\int_0^t x(t')\sin\omega_k t'\,dt' + \frac{2C_kR_k'}{m_k\omega_k}\int_0^t x(t')\sin\omega_k(t-t')dt' \\
&-\frac{2C_k^2}{m_k^2\omega_k^2}\int_0^t dt'\int_0^{t'} dt''\,x(t')x(t'')\sin\omega_k(t-t')\sin\omega_k t''\Big].
\end{aligned}$$

averaged over

$$\rho_R(\mathbf{R}', \mathbf{Q}', 0) = \prod_k \rho_R^{(k)}(R'_k, Q'_k, 0) =$$

$$= \prod_k \frac{m_k\omega_k}{2\pi\hbar \sinh\left(\frac{\hbar\omega_k}{k_B T}\right)} \exp - \left\{ \frac{m_k\omega_k}{2\hbar \sinh\left(\frac{\hbar\omega_k}{k_B T}\right)} \left[\left({R'_k}^2 + {Q'_k}^2 \right) \cosh\left(\frac{\hbar\omega_k}{k_B T} \right) - 2R'_k Q'_k \right] \right\}$$

Resulting super-propagator

$$\mathcal{J}(x, y, t; x', y', 0) = \int_{x'}^{x} \int_{y'}^{y} \mathcal{D}x(t')\mathcal{D}y(t') \exp \frac{i}{\hbar} \Big\{ \tilde{S}_0[x(t')] - \tilde{S}_0[y(t')] -$$

$$- \int_0^t \int_0^\tau d\tau d\sigma [x(\tau) - y(\tau)] \alpha_I(\tau - \sigma)[x(\sigma) + y(\sigma)] \Big\} \times$$

$$\times \exp - \frac{1}{\hbar} \int_0^t \int_0^\tau d\tau d\sigma [x(\tau) - y(\tau)] \alpha_R(\tau - \sigma)[x(\sigma) - y(\sigma)] \Big\}$$

Time kernels

$$\alpha_R(\tau-\sigma)=\sum_k \frac{C_k^2}{2m_k\omega_k}\coth\frac{\hbar\omega_k}{2k_BT}\cos\omega_k(\tau-\sigma)$$

$$\alpha_I(\tau-\sigma)=-\sum_k \frac{C_k^2}{2m_k\omega_k}\sin\omega_k(\tau-\sigma)$$

Under the previous choice of the spectral function the final super-propagator is

$$\mathcal{J}(x,x',t;y,y',0)=\int_{x'}^{x}\int_{y'}^{y}\mathcal{D}x(t')\mathcal{D}y(t')$$

$$\times\exp\frac{i}{\hbar}\left\{S_0[x(t')]-S_0[y(t')]-M\gamma\int_0^t(x\dot{x}-y\dot{y}+x\dot{y}-y\dot{x})dt'\right\}$$

$$\times\exp-\frac{2M\gamma}{\pi\hbar}\int_0^{\Omega}d\omega\,\omega\,\coth\frac{\hbar\omega}{2k_BT}\times\int_0^t\int_0^\tau d\tau d\sigma[x(\tau)-y(\tau)]\cos\omega(\tau-\sigma)[x(\sigma)-y(\sigma)]$$

The equilibrium reduced density operator

Full operator $\langle x\mathbf{R}|e^{-\beta H}|y\mathbf{Q}\rangle = \rho(x,\mathbf{R};x,\mathbf{Q},\beta)$

Path integral representation

$$\rho(x,\mathbf{R};y,\mathbf{Q},\beta) = \int_y^x \int_{\mathbf{Q}}^{\mathbf{R}} \mathcal{D}q(\tau)\mathcal{D}\mathbf{R}(\tau)\exp - \frac{1}{\hbar} S_E[q(\tau),\mathbf{R}(\tau)]$$

Euclidean (imaginary – time) action of the complete system

$$S_E[q(\tau),\mathbf{R}(\tau)] = \int_0^{\hbar\beta} d\tau \Big\{ \frac{1}{2}M\dot{q}^2 + V(q) +$$

$$+ \sum_k \Big(C_k q R_k + \frac{1}{2} m_k \dot{R}_k^2 + \frac{1}{2} m_k \omega_k^2 R_k^2 + \frac{C_k^2 q^2}{2 m_k \omega_k^2} \Big) \Big\}$$

Reduced density operator of the system

$$\tilde{\rho}(x,y,\beta) \equiv \int d\mathbf{R}\, \rho(x,\mathbf{R};y,\mathbf{R},\beta) = \int_y^x \mathcal{D}q(\tau) \int d\mathbf{R} \int_{\mathbf{R}}^{\mathbf{R}} \mathcal{D}\mathbf{R}(\tau) \exp - \frac{1}{\hbar} S_E[q(\tau),\mathbf{R}(\tau)]$$

Final form

$$\tilde{\rho}(x,y,\beta) = \tilde{\rho}_0(\beta) \int_y^x \mathcal{D}q(\tau)\ \exp - \frac{1}{\hbar} S_{eff}[q(\tau)]$$

Effective euclidean action

$$S_{eff}[q(\tau)] = \int_0^{\hbar\beta} d\tau \left\{ \frac{1}{2} M\dot{q}^2 + V(q) \right\} + \frac{1}{2} \int_{-\infty}^{+\infty} d\tau' \int_0^{\hbar\beta} d\tau \alpha(\tau-\tau') \{q(\tau) - q(\tau')\}^2$$

Imaginary time kernel

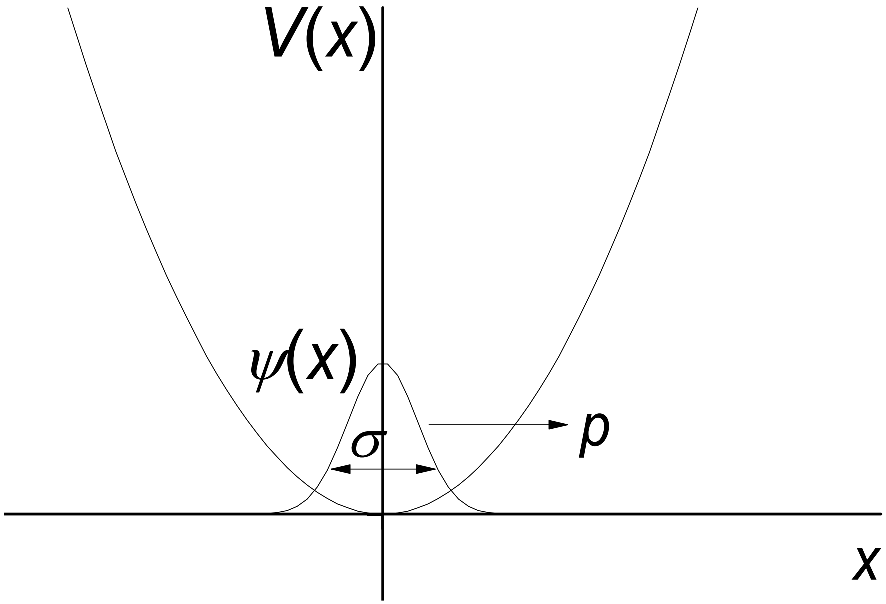
$$\alpha(\tau-\tau') \equiv \sum_k \frac{C_k^2}{4m_k\omega_k} \exp - \omega_k|\tau-\tau'| = \frac{1}{2\pi} \int_0^\infty d\omega\, J(\omega) \exp - \omega_k|\tau-\tau'|$$

Ohmic dissipation

$$\alpha(\tau-\tau') = \frac{1}{2\pi} \int_0^\infty d\omega\, \eta\omega \exp - \omega_k|\tau-\tau'| = \frac{\eta}{2\pi} \frac{1}{(\tau-\tau')^2}$$

C. Applications

Damped harmonic oscillator (real time)



$$\psi(x') = \frac{1}{(2\pi\sigma^2)^{1/4}} \exp \frac{ipx'}{h} \exp - \frac{x'^2}{4\sigma^2}$$

Initial state

A.O.C. & A. J. Leggett
Physica A 121(3), 587 (1983)

Time evolution

$$\tilde{\rho}(x,x,t) = \left(\frac{1}{2\pi\sigma^2(t)}\right)^{1/2} \exp -\frac{1}{2\sigma^2(t)}\,(x - x_0(t))^2$$

Center of the wavepacket
($\gamma \equiv \frac{\eta}{2M}$ and $\omega \equiv \sqrt{\omega_0^2 - \gamma^2}$)

$$x_0(t) = \frac{p}{M\omega}\sin\omega t\; e^{-\gamma t}$$

Width at equilibrium

$$\sigma^2(\infty) = \frac{\hbar}{\pi}\int_0^\infty d\nu \coth\frac{\hbar\nu}{2kT}\left(\frac{1}{M}\frac{2\gamma\nu}{\left(\omega_0^2 - \nu^2\right)^2 + 4\gamma^2\nu^2}\right)$$

Fluctuation-dissipation theorem

$$\sigma^2(\infty) = \frac{\hbar}{\pi}\int_0^\infty d\nu \coth\frac{\hbar\nu}{2kT}\chi''(\nu)$$

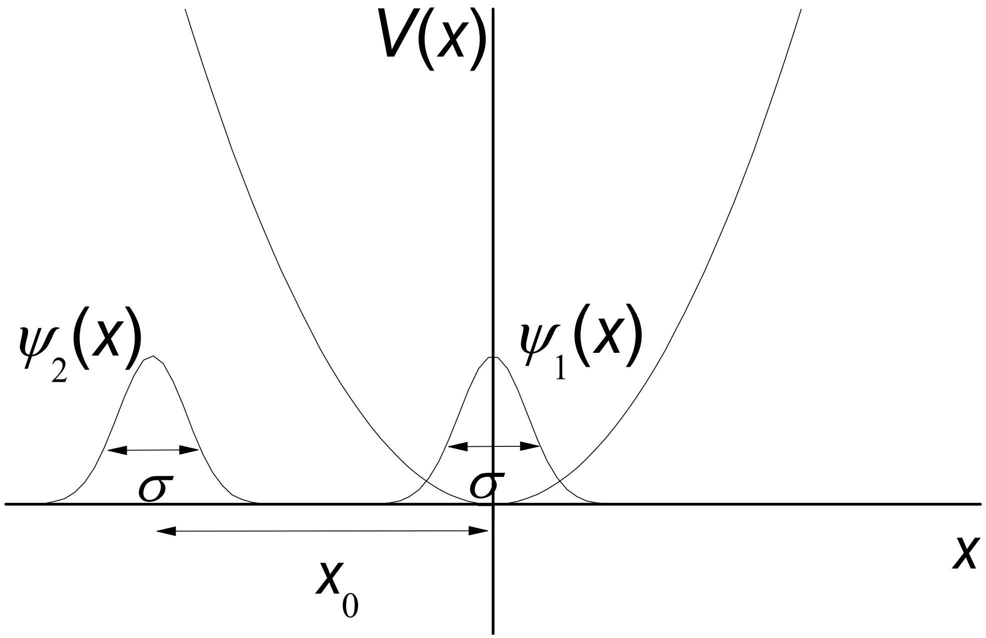
Behavior of the width at zero temperature for any value of the damping constant

$$\sigma^2(\infty) = \frac{\hbar}{2M\omega_0} f(\alpha) \qquad \left(\alpha \equiv \frac{\gamma}{\omega_0}\right)$$

$$f(\alpha) = \begin{cases} \frac{1}{\sqrt{1-\alpha^2}} \left(1 - \frac{2}{\pi} \tan^{-1} \frac{\alpha}{\sqrt{1-\alpha^2}}\right) & \text{se} \quad \alpha < 1 \\ \frac{1}{\sqrt{\alpha^2-1}} \frac{1}{\pi} \ln \left| \frac{\alpha+\sqrt{\alpha^2-1}}{\alpha-\sqrt{\alpha^2-1}} \right| & \text{se} \quad \alpha > 1 \end{cases}$$

Finite damping always reduces the width

Decoherence (real time)



Initial state

$$\psi(x) = \psi_1(x) + \psi_2(x) = \tilde{\mathcal{N}} \left[\exp - \frac{x^2}{4\sigma^2} + \exp - \frac{(x-q_0)^2}{4\sigma^2} \right]$$

A.O.C. & A. J. Leggett
Phys. Rev A 31, 1059 (1985)

Initial density operator

$$\rho(x', y', 0) = \rho_1(x', y', 0) + \rho_2(x', y', 0) + \rho_{int}(x', y', 0)$$

Time evolution $$\tilde{\rho}(x,t) = \tilde{\rho}_1(x,t) + \tilde{\rho}_2(x,t) + \tilde{\rho}_{int}(x,t)$$

Linearity of the time evolution

$$\tilde{\rho}_{int}(x,x,t) = \int\int dx'dy'\, \mathcal{J}(x,x,t;x',y',0)\, \tilde{\rho}_{int}(x',y',0)$$

$$\tilde{\rho}_{int}(x,t) = 2\sqrt{\tilde{\rho}_1(x,t)}\sqrt{\tilde{\rho}_2(x,t)}\cos\phi(x,t)\exp -f(t)$$

Attenuation factor $$\boxed{\exp -f(t) \approx \exp -\Gamma t}$$

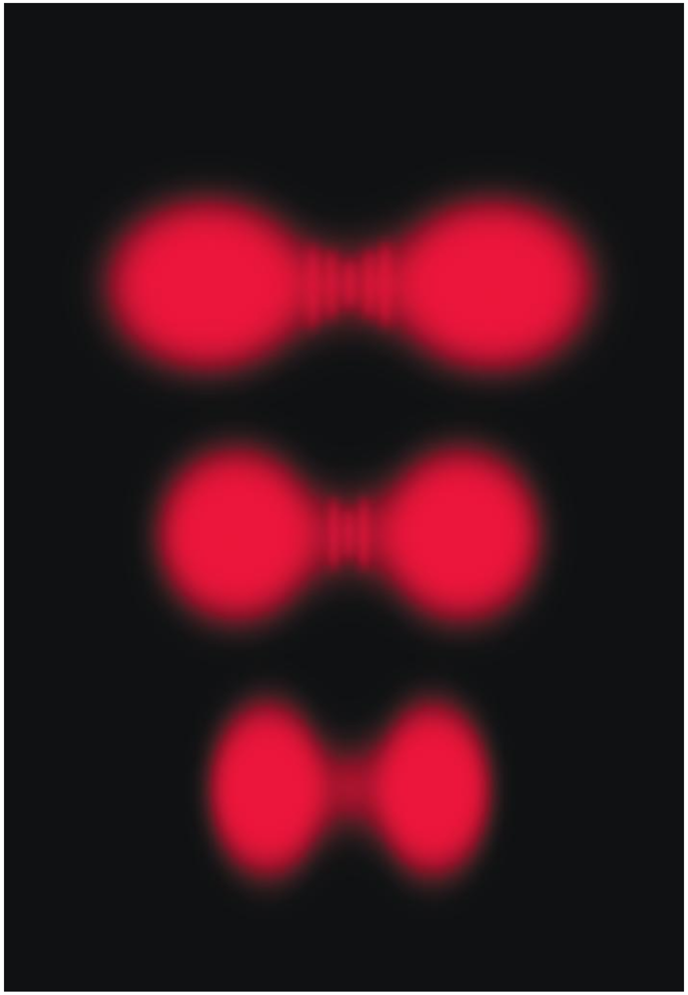
Decoherence rate

$$\Gamma = \begin{cases} \text{high temperatures} \quad (\kappa \ll 1) \begin{cases} \frac{2NkT}{\hbar\omega_0}\gamma & \text{if} \quad \gamma \ll \omega_0 \\ \frac{2NkT}{\hbar\omega_0}\frac{\omega^2}{2\gamma} & \text{if} \quad \gamma \gg \omega_0 \end{cases} \\ \\ \text{low temperatures} \quad (\kappa \gg 1) \begin{cases} N\gamma & \text{if} \quad \gamma \ll \omega_0 \\ N\frac{\omega_0^2}{2\gamma} & \text{if} \quad \gamma \gg \omega_0 \end{cases} \end{cases}$$

$N \equiv x_0^2/4\sigma^2$ is also the average number of energy quanta initially in the system

$\kappa \equiv \hbar\omega_0/kT$ is an inverse dimensionless temperature

Decoherence: Supression of the interference between wavepackets of a harmonic oscillator in a dissipative medium



Subtle procedures of preparation of quantum mechanical states allow us to build the following "Schrödinger cat" state either in optical cavities where one has the superposition of two coherent states of the electromagnetic field

$$|\psi\rangle = |\alpha\rangle + |-\alpha\rangle$$

Brune M. et al PRL 77, 4887 (1996)

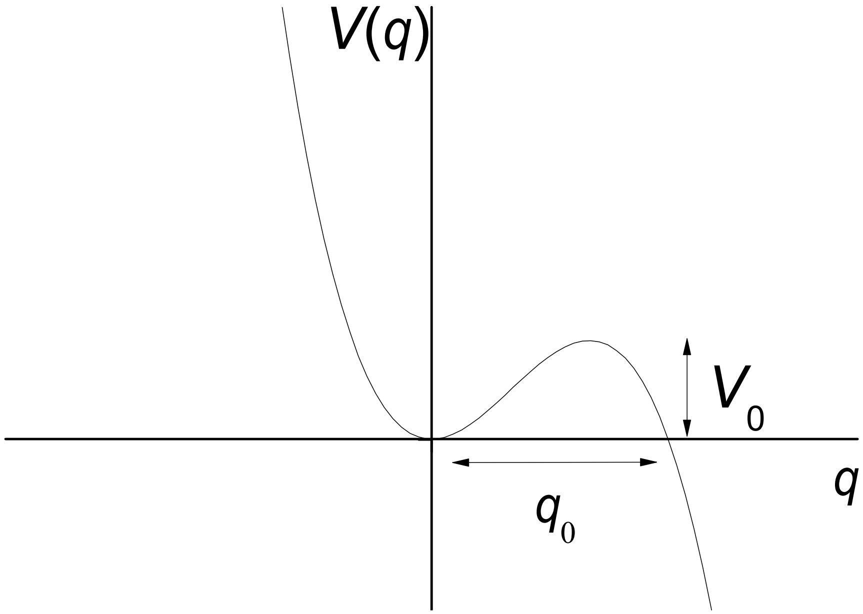
or in atomic traps where

$$|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$$

Monroe C. et al Science 272, 1131 (1996)

represents a superposition of two spatially separated states of the ion, exactly like we proposed in the case of the harmonic oscillator

Dissipative quantum tunneling (imaginary time)



$$V(q) = \frac{1}{2} M\omega_0^2 q^2 - \lambda q^3 \qquad (\lambda > 0)$$

One needs the imaginary part of the energy of the full system

$$\rho_{eq}(x, \mathbf{R}; y, \mathbf{Q}, \beta) = \sum_n \psi_n(x, \mathbf{R})\psi_n^*(y, \mathbf{Q}) \exp -\beta E_n$$

$$\tilde{\rho}_{eq}(x, y, \beta) = \int d\mathbf{R} \rho_{eq}(x\mathbf{R}; y\mathbf{R}) = \int d\mathbf{R} \sum_n \psi_n(x, \mathbf{R})\psi_n^*(y, \mathbf{R}) \exp -\beta E_n$$

Path integral representation

$$\tilde{\rho}_{eq}(x,y,\beta) = \int d\mathbf{R} \langle x\mathbf{R}|e^{-\beta H}|y\mathbf{R}\rangle$$
$$= \int d\mathbf{R} \int_{y,\mathbf{R}}^{x,\mathbf{R}} \mathcal{D}q(\tau')\mathcal{D}\mathbf{R}(\tau') \exp -\frac{S_E}{\hbar}[q(\tau'),\mathbf{R}(\tau')]$$

Low temperature behavior

$$\int d\mathbf{R} |\psi_0(0,\mathbf{R})|^2 e^{-\tau E_0/h} \approx \int d\mathbf{R} \int_{0,\mathbf{R}}^{0,\mathbf{R}} \mathcal{D}q(\tau')\mathcal{D}\mathbf{R}(\tau') \exp -\frac{S_E}{\hbar}[q(\tau'),\mathbf{R}(\tau')]$$

Final form

$$\tilde{\rho}(0,0,\beta) = \tilde{\rho}_0(\beta) \int_0^0 \mathcal{D}q(\tau') \exp -\frac{S_{eff}}{\hbar}[q(\tau')]$$

Effectice euclidean action

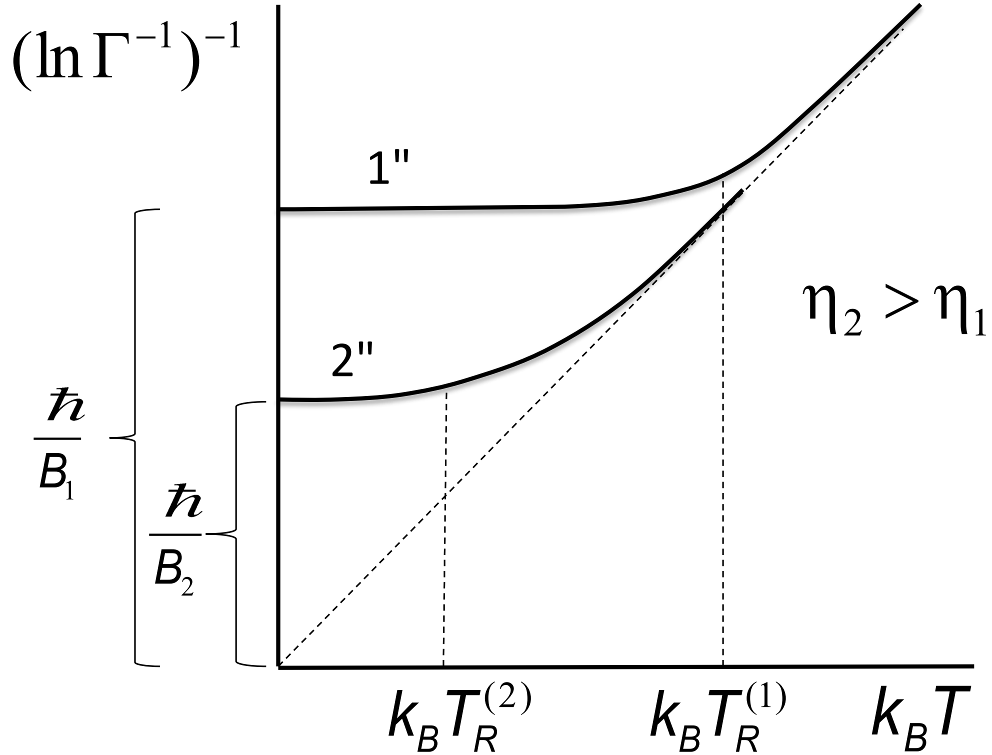
$$S_{eff}[q(\tau')] = \int_0^{\tau} \left\{ \frac{1}{2} M \dot{q}^2 + V(q) \right\} d\tau' + \frac{\eta}{4\pi} \int_{-\infty}^{\infty} dt'' \int_0^{\tau'} d\tau' \frac{\{q(\tau') - q(\tau'')\}^2}{(\tau' - \tau'')^2}$$

Decay rate

$$\Gamma = A \exp - \frac{B}{\hbar}$$

Effectice euclidean action at the bounce solution

$$B \equiv S_{eff}[q_c] = \int_{-\infty}^{\infty} \left\{ \frac{1}{2} M \ddot{q}_c^2 + V(q_c) \right\} d\tau' + \frac{\eta}{4\pi} \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau'' \frac{\{q_c(\tau') - q_c(\tau'')\ \}^2}{(\tau' - \tau'')^2}$$



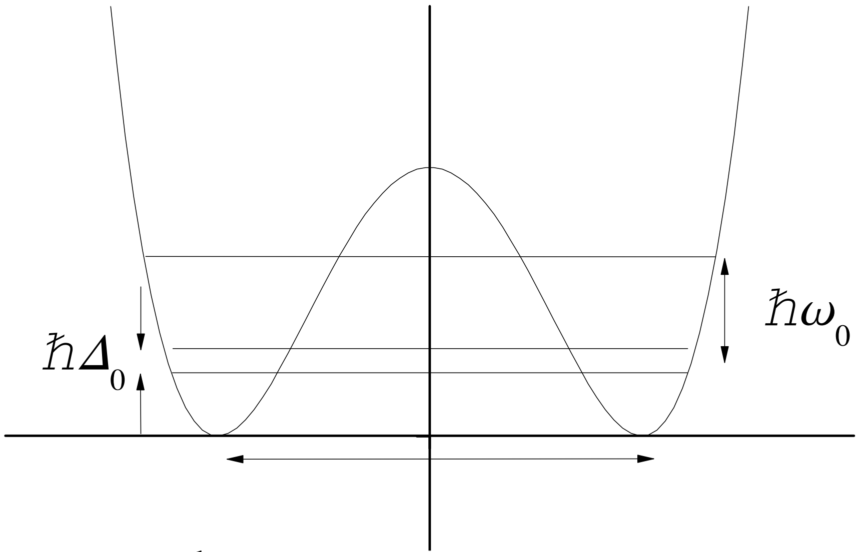
Correction for low damping $$\Delta B = \frac{45}{\pi^3}\zeta(3)\alpha B_0 = \frac{12}{\pi^3}\zeta(3)\eta q_0^2$$

Action for strong damping $$B = \frac{2\pi}{9}\eta q_0^2 + \mathcal{O}\left(\frac{\omega_0}{\gamma}\right)$$

Classical decay rate (Arrenhius) $$\Gamma_\beta \approx A_{cl}(\beta)\exp -\beta V_0$$

A.O.C. & A. J. Leggett
Annals of Phys. 149, 374 (1983)

Dissipative quantum coherence (imaginary + real time)



Quartic potential

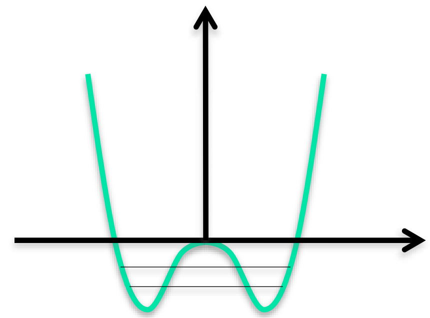
$$V(q) = -\frac{1}{2}m\omega_0^2 q^2 + \frac{\lambda}{4}q^4 \;\; (\lambda > 0)$$

Two lowest lying states $\psi_E = \frac{1}{\sqrt{2}}(\psi_R + \psi_L)$ and $\psi_O = \frac{1}{\sqrt{2}}(\psi_R - \psi_L)$

Energy splitting $\hbar\Delta_0 = E_O - E_E \propto \langle\psi_R|\mathcal{H}|\psi_L\rangle = \langle\psi_L|\mathcal{H}|\psi_R\rangle$

If $\langle q(0)\rangle = \frac{q_0}{2}$ ➡ $\langle q(t)\rangle = \frac{q_0}{2}\cos\Delta_0 t$

Quartic Hamiltonian



$$H = \frac{p^2}{2m} - \frac{1}{2} m\omega_0^2 q^2 + \frac{\lambda}{4} q^4 + q \sum_k C_k \, q_k + \sum_k \frac{p_k^2}{2m_k} + \sum_k \frac{1}{2} \, m_k \, \omega_k^2 \, q_k^2$$

truncation

A. J. Leggett *et al.*
Rev. Mod. Phys. 59, 1 (1987)

Spin – boson Hamiltonian

$$H = -\frac{1}{2} \hbar \Delta \sigma_x + \frac{1}{2} \epsilon \, \sigma_z + \frac{1}{2} \, q_0 \, \sigma_z \sum_k C_k \, q_k + \sum_k \frac{p_k^2}{2m_k} + \sum_k \frac{1}{2} \, m_k \, \omega_k^2 \, q_k^2$$

Spin – boson analogous to NMR: Bloch equations $(\epsilon = 0)$

$$\frac{dS_x}{dt} = -\frac{S_x - S_x^{eq}}{T_1}$$

$$\frac{dS_y}{dt} = \Delta S_z - \frac{S_y}{T_2}$$

$$\frac{dS_z}{dt} = -\Delta S_y$$

$$P(t) \equiv \langle S_z(t) \rangle$$

$$\frac{d^2 P}{dt^2} + \frac{1}{T_2}\frac{dP}{dt} + \Delta^2 P = 0$$

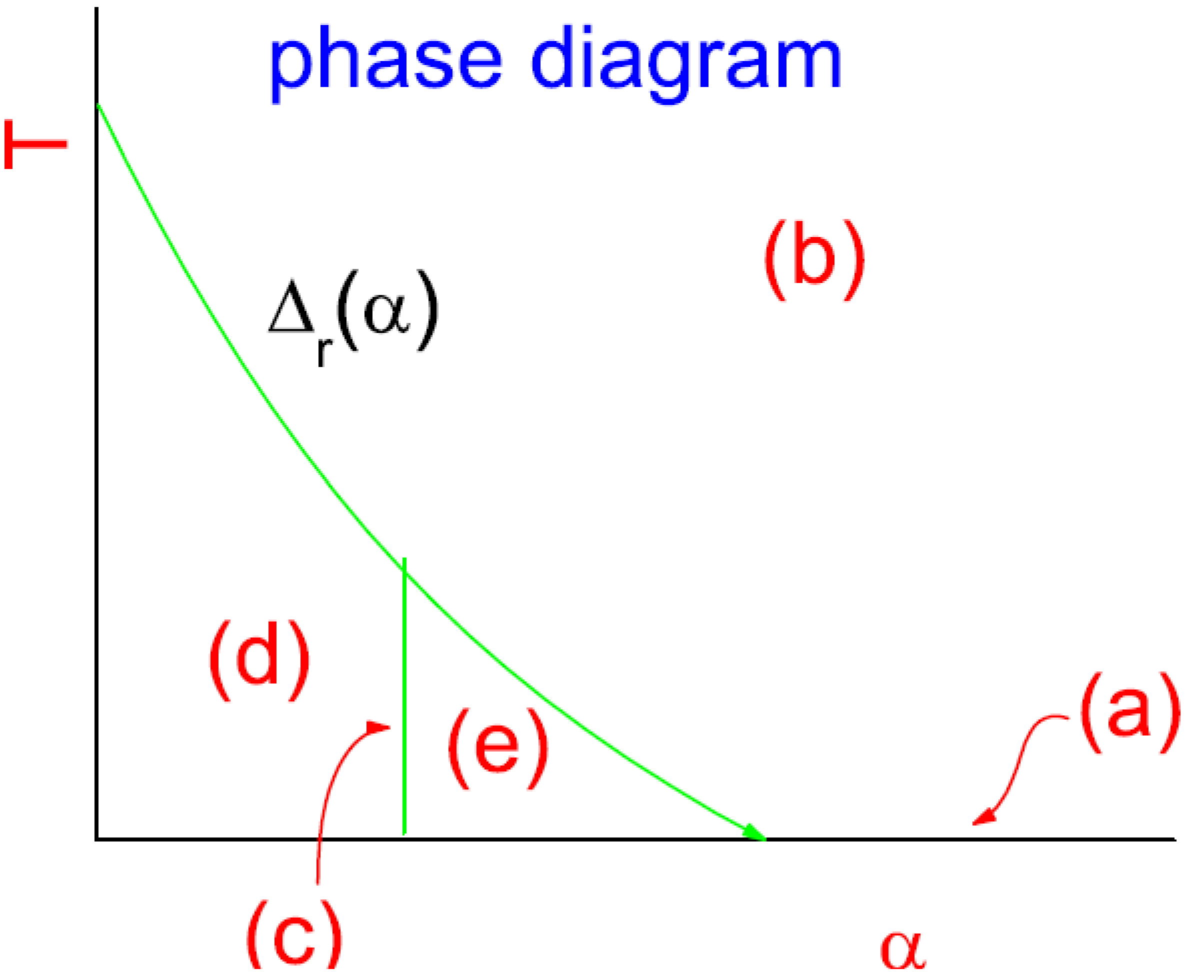
Perturbation theory in the spin – boson model

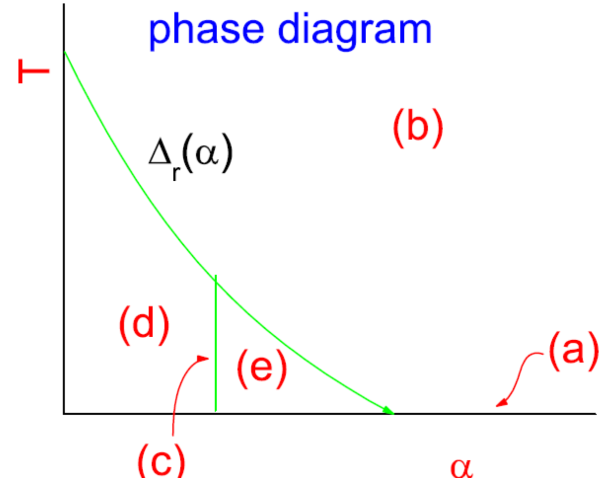
$$\frac{1}{T_1} = \frac{1}{T_2} = \frac{q_0^2}{2\hbar} J(\Delta) \coth \frac{\beta\hbar\Delta}{2}$$

In general $2T_1 > T_2$

Solutions

$$\alpha \equiv \frac{\eta q_0^2}{2\pi\hbar} \qquad \text{and} \qquad \Delta_r \equiv \Delta\left[\frac{\Delta}{\Omega}\right]^{\frac{\alpha}{(1-\alpha)}}$$

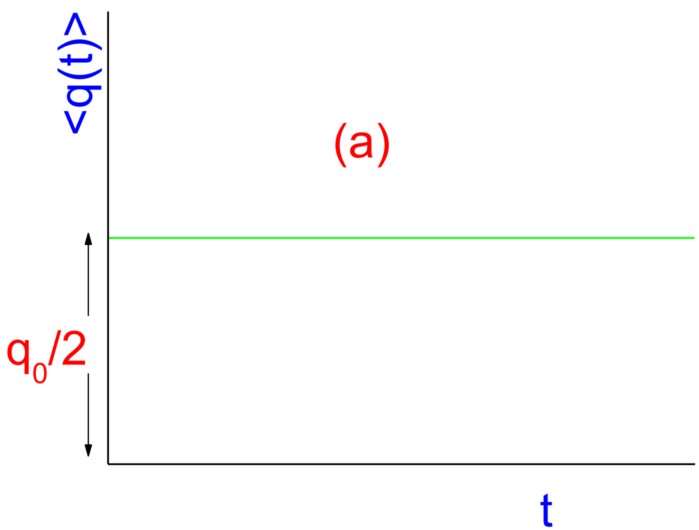


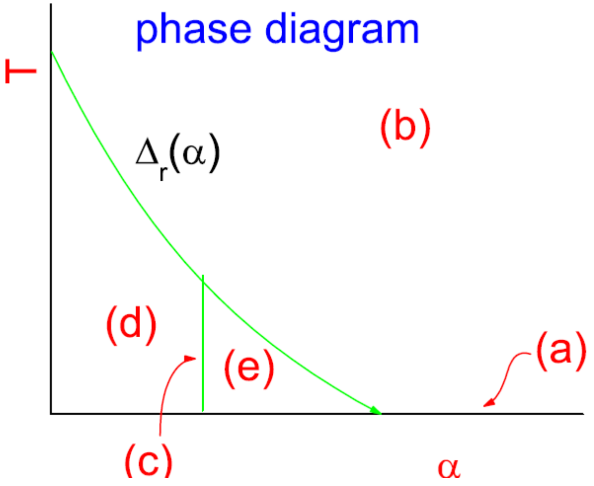


Dimensionless dissipation and renormalized splitting

$$\alpha \equiv \frac{\eta q_0^2}{2\pi\hbar} \qquad \text{and} \qquad \Delta_r \equiv \Delta\left[\frac{\Delta}{\Omega}\right]^{\frac{\alpha}{(1-\alpha)}}$$

$\alpha \geq 1$ and $T = 0$ ⟹ Localization

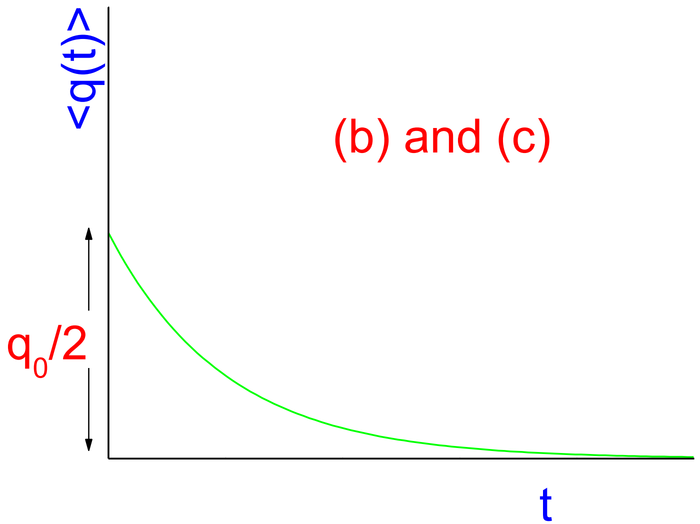




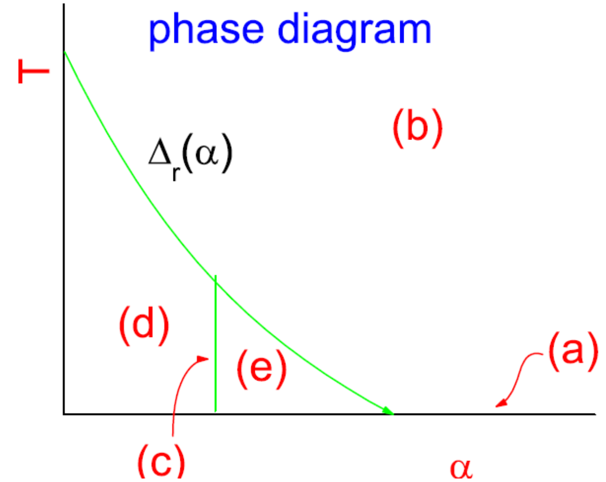
Strong damping and/or high temperatures $\alpha k_B T/\hbar \gg \Delta_r$

Exponential decay

$$P(t) = \exp - \frac{t}{\tau} \qquad \tau^{-1} = \frac{\sqrt{\pi}}{2} \frac{\Gamma(\alpha)}{\Gamma(\alpha + \frac{1}{2})} \frac{\Delta_r^2}{k_B T/\hbar} \left[\frac{\pi k_B T}{\hbar \Delta_r} \right]^{2\alpha}$$

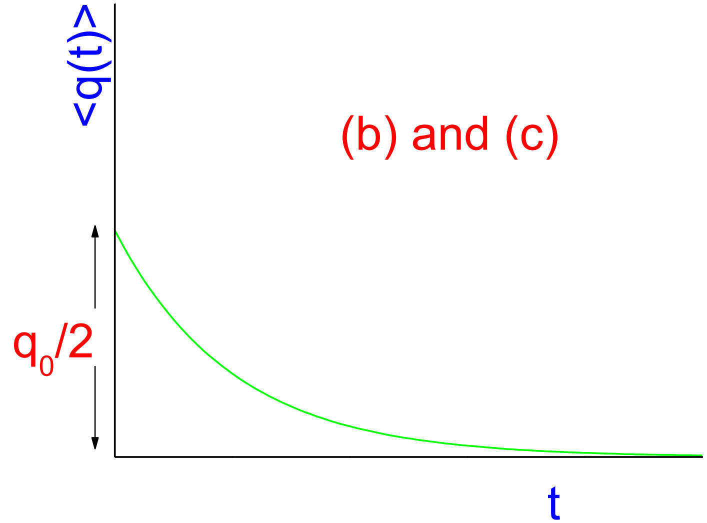


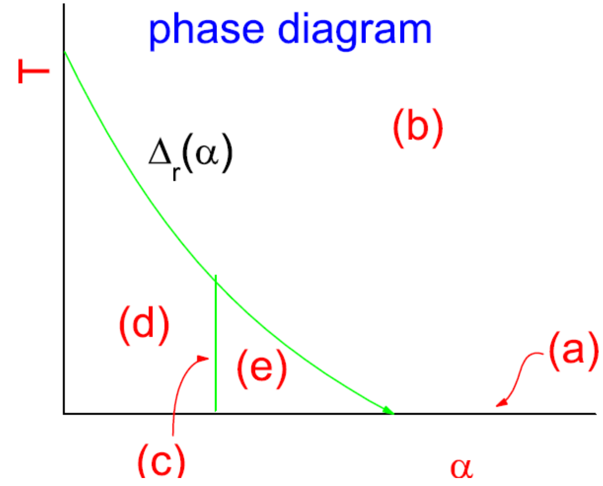
Exactly soluble model $\alpha = 1/2$



Exponential decay

$$P(t) = \exp\Big[-\frac{\pi}{2}\frac{\Delta^2}{\Omega}t\Big]$$





Weak damping at zero temperature

$T = 0$ and $0 \le \alpha < 1/2$

↓

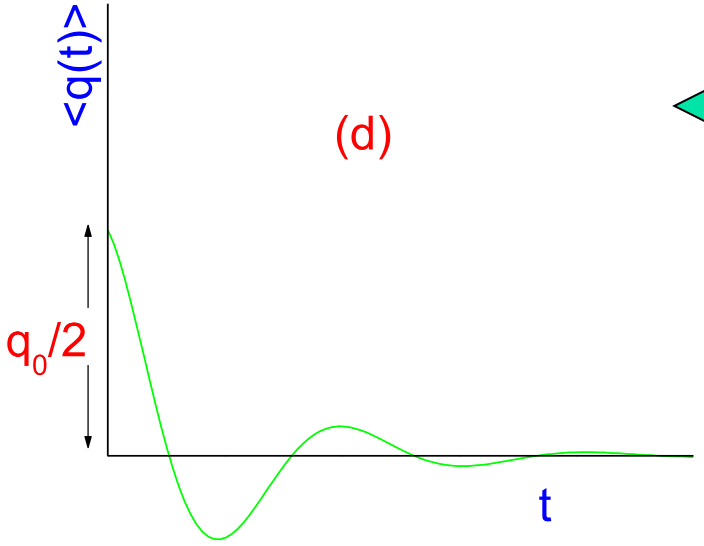
Oscillatory decay

$$\Delta_{eff} \equiv [\Gamma(1-2\alpha)\cos\pi\alpha]^{\frac{1}{2(1-\alpha)}}\Delta_r$$

$$P(y) = P_{coh}(y) + P_{inc}(y) \qquad \text{where} \quad y \equiv \Delta_{eff} t$$

$$P_{coh}(y) \equiv \frac{1}{1-\alpha}\cos\left\{\left[\cos\left(\frac{\pi}{2}\frac{\alpha}{1-\alpha}\right)\right]y\right\}\exp-\left\{\left[\sin\left(\frac{\pi}{2}\frac{\alpha}{1-\alpha}\right)\right]y\right\}$$

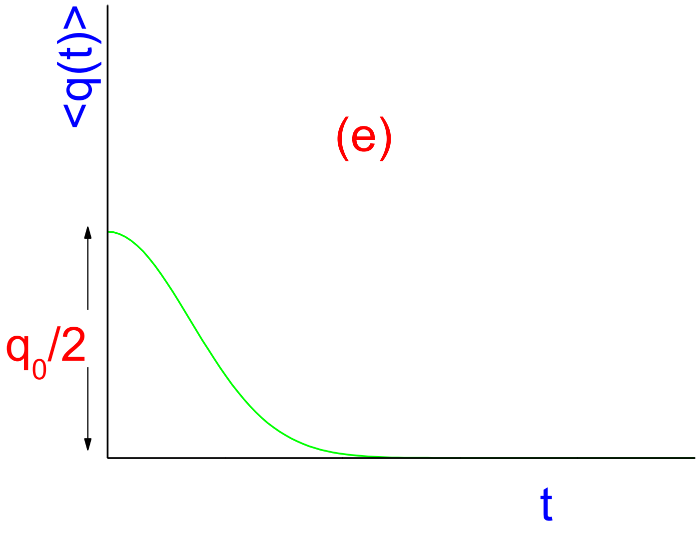
$$P_{inc}(y) \equiv -\frac{\sin 2\pi\alpha}{\pi}\int_0^\infty dz\,\frac{z^{2\alpha-1}e^{-zy}}{z^2+2z^{2\alpha}\cos 2\pi\alpha + z^{4\alpha-2}}$$



Oscillatory decay

Incoherent relaxation

$T = 0$ and $1/2 \leq \alpha < 1$



D. Collision model (quantum)

For *fermionic* or *bosonic* environments

$$\mathcal{H} = \frac{p^2}{2M} + \sum_{i=1}^{N} U(q - q_i) + \sum_{i=1}^{N} \frac{p_i^2}{2m}$$

Performing the unitary (translation) transformation

$$\mathcal{U} = \exp -\frac{i}{\hbar}\sum_{j=1}^{N} p_j q$$

one has $\mathcal{H}' = \mathcal{U}\mathcal{H}\mathcal{U}^{-1} = \frac{1}{2M}\left(p - \sum_{j=1}^{N} p_j\right)^2 + \sum_{j=1}^{N}\left(\frac{p_j^2}{2m} + U(q_j)\right)$

In second quantized form

$$\mathcal{H}' = \frac{1}{2M}\left(p - \sum_{i,j} \hbar g_{ij} a_i^\dagger a_j\right)^2 + \sum_i (\hbar\Omega_i - \mu) a_i^\dagger a_i$$

where $g_{ij} = \frac{1}{\hbar}\langle i|p'|j\rangle$

Gaussian approximation $\mathcal{F}[x,y] = \exp -\frac{i}{\hbar}\Phi_I[x,y] \exp -\frac{1}{\hbar}\Phi_R[x,y]$

where
$$\begin{cases} \Phi_I = \int_0^t dt' \int_0^{t'} dt''(\dot{x}(t') - \dot{y}(t'))\hbar\Gamma_I(t'-t'')(\dot{x}(t'') + \dot{y}(t'')) \\ \Phi_R = \int_0^t dt' \int_0^{t'} dt''(\dot{x}(t') - \dot{y}(t'))\hbar\Gamma_R(t'-t'')(\dot{x}(t'') - \dot{y}(t'')) \end{cases}$$

with
$$\begin{cases} \Gamma_R(t) = \frac{1}{2}\sum_{i,j} |g_{ij}|^2(\overline{n}_i + \overline{n}_j \pm 2\overline{n}_i\overline{n}_j)\cos(\Omega_i - \Omega_j)t \\ \Gamma_I(t) = \frac{1}{2}\sum_{i,j} |g_{ij}|^2(\overline{n}_i - \overline{n}_j)\sin(\Omega_i - \Omega_j)t \end{cases}$$

which with the help of $S(\omega,\omega') = \sum_{i,j} |g_{ij}|^2\delta(\omega - \Omega_i)\delta(\omega' - \Omega_j)$

allows us to define

$$\gamma(t) = -\frac{\hbar}{M}\frac{d\Gamma_I(t)}{dt} = -\frac{\hbar}{2M}\int d\omega \int d\omega' S(\omega,\omega')[\overline{n}(\omega) - \overline{n}(\omega')](\omega - \omega')\cos(\omega - \omega')t$$

$$D(t) = -\hbar\frac{d^2\Gamma_R}{dt^2} = \frac{\hbar^2}{2}\int d\omega \int d\omega' S(\omega,\omega')[\overline{n}(\omega) + \overline{n}(\omega') \pm 2\overline{n}(\omega)\overline{n}(\omega')](\omega - \omega')^2\cos(\omega - \omega')t$$

For a repulsive contact potential $U(q - q_i) = V_0 a\delta(q - q_i)$
we have in the long time approximation

$$\gamma(t) = \bar{\gamma}(T)\delta(t) \quad \text{with} \quad \bar{\gamma}(T) = -\frac{m}{2\pi\hbar M}\int dE\, E\, R(E)\,\frac{d\overline{n}}{dE}$$

and $R(E) = \dfrac{E_0}{E_0 + E}$ with $E_0 = \dfrac{mV_0^2 a^2}{2\hbar^2}$

A.O.C. & A. H. Castro Neto
Phys. Rev B 52, 4198 (1995)

in the fermionic case but they present no special difficulty either. It is a very easy task to show that the occupation number for bosons has the following assymptotic limits:

fermions

$$\bar{n}(E) = \frac{k_BT}{E+|\mu|}\,, \quad E+|\mu| \ll k_BT,$$

$$\bar{n}(E) = e^{-\frac{E+|\mu|}{k_BT}}\,, \quad E+|\mu| \gg k_BT. \qquad (4.23)$$

Observe that for bosons the chemical potential μ is always negative.[17]

Now, if we assume that, when $T \to 0$, $\mu(T)$ behaves as[27]

fermions

lowering its zero temperature value.

The classical limit, which we define as $k_BT \gg E_F$, ca[n] also be analyzed if we use that in this limit the Ferm[i] occupation number is the same as in the distinguishabl[e] particle case, namely,[17]

Once again following our procedure in the fermionic case we can also study distinct temperature dependences of the damping constant as we show below.

bosons

(a) $E_0 \gg k_BT$; here $\bar{\gamma}$ vanishes at $T = 0$ and increases with temperature as in (4.33). When $k_BT \gg E_0 \gg k_BT_0$ it decays as $T^{-1/2}$ and, consequently, it must present a maximum at intermediate temperatures ($E_0 \gg k_BT \gg k_BT_0$).

(b) $E_0 \to \infty$; the low temperature behavior is the same as before but now one can never go beyond the regime $E_0 \gg k_BT \gg k_BT_0$ and therefore $\bar{\gamma}$ behaves like $T^{1/2}$.

Notice that we have not analyzed above the condition $k_BT_0 \gg E_0$ because it does not make sense for bosons. In this case the strength of the barrier is below the minu-

But RG perturbation theory for the 1 – D fermionic bath

$$\bar{\gamma}(T) \sim T^4$$

Gaussian approximation is not appropriate

A. H. Castro Neto & M.P.A.Fisher
Phys. Rev B 52, 4198 (1995)

Motion of topological excitations (solitons, vortices, etc)

Alternative way relies on the fact that given $\mathcal{L}(x,t) = \frac{1}{2}\left(\dot{\phi}^2 - {\phi'}^2\right) - U(\phi)$

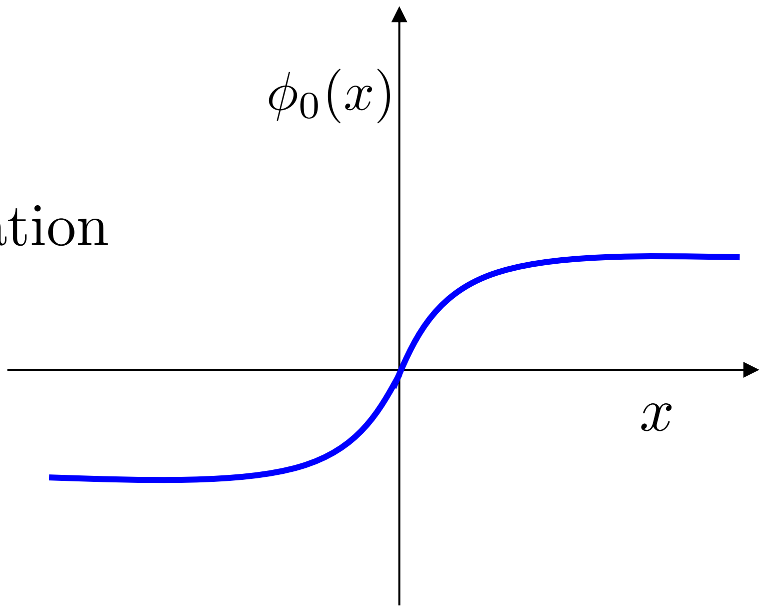
Localized solution of $\ddot{\phi} - \phi'' = -\frac{\partial U(\phi)}{\partial \phi}$ behaves as a particle in a medium

Boundary conditions $\lim_{x\to\pm\infty} \phi_0(x) = \text{minima of } U$

Solution

$$\phi'' = \frac{\partial U(\phi)}{\partial \phi} \quad + \quad \text{Lorentz transformation}$$

$$x - x_0 = \pm \int_{\phi_0(x_0)}^{\phi_0(x)} \frac{d\phi}{\sqrt{2U(\phi)}}$$



Collective coordinate method

$$\phi(x,t) = \phi_0(x - X(t)) + \sum_{i=1}^{\infty} q_i(t)\eta_i(x - X(t))$$

Second functional derivative at $\phi_0(x)$:

$$\left[-\frac{d^2}{dx^2} + \left.\frac{d^2U}{d\phi^2}\right|_{\phi_0}\right]\eta_i(x) = \omega_i^2\eta_i(x)$$

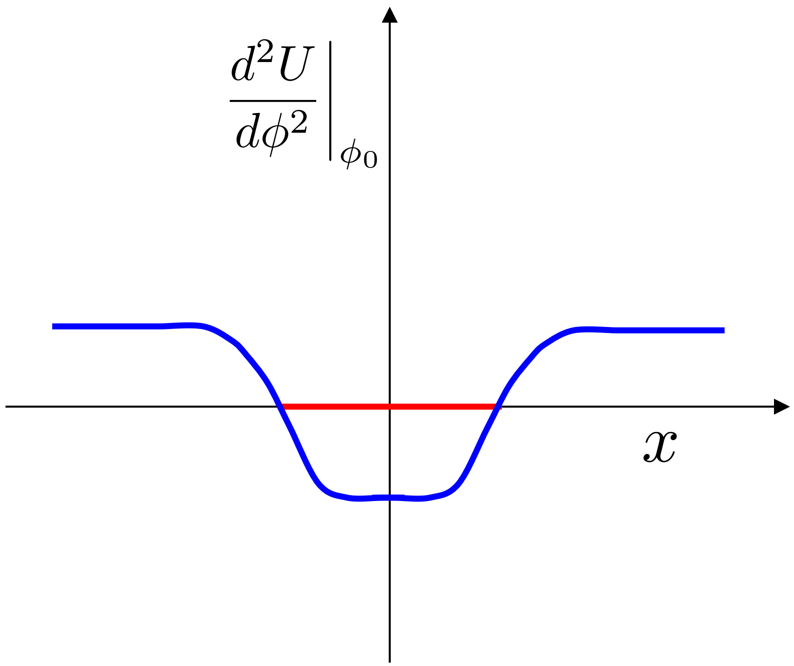
$$\widehat{H} = \frac{1}{2M}\left(\widehat{\mathbf{P}} - \widehat{\mathbf{P}}_{\text{env}}\right)^2 + \sum_m \hbar\omega_m b_m^\dagger b_m$$

$$\widehat{\mathbf{P}}_{\text{env}} = \sum_{mn} \mathbf{D}_{mn} b_m^\dagger b_n + \sum_{mn} \mathbf{C}_{mn}\left(b_m b_n - b_m^\dagger b_n^\dagger\right)$$

$$\mathbf{D}_{mn} = \frac{1}{2}\left[\sqrt{\left(\frac{\omega_n}{\omega_m}\right)} + \sqrt{\left(\frac{\omega_m}{\omega_n}\right)}\right]\mathbf{G}_{mn}$$

$$\mathbf{C}_{mn} = \frac{1}{4}\left[\sqrt{\left(\frac{\omega_n}{\omega_m}\right)} - \sqrt{\left(\frac{\omega_m}{\omega_n}\right)}\right]\mathbf{G}_{mn}$$

$$\mathbf{G}_{mn} = -\frac{i}{\hbar}\int_{-\infty}^{\infty} \eta_m(\mathbf{x})(\nabla\eta_n(\mathbf{x}))d\mathbf{x}$$

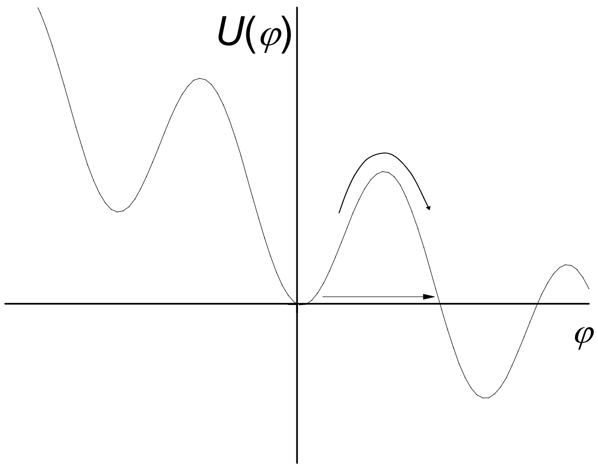


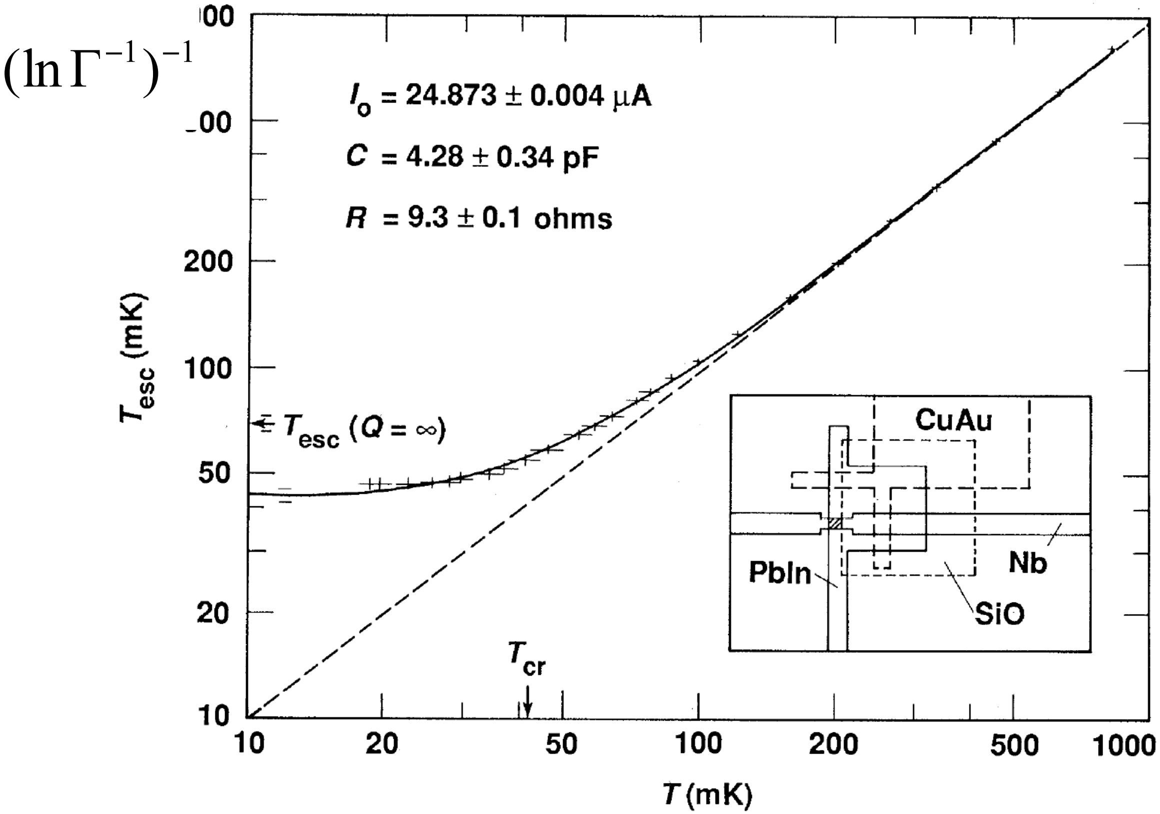
For bright solitons in BECs
D. K. Efimov, J. Hofmann & V. Galitski
Phys. Rev. Lett. 116, 225301 (2016)

Lecture 3

A. Experimental realizations

CBJJs

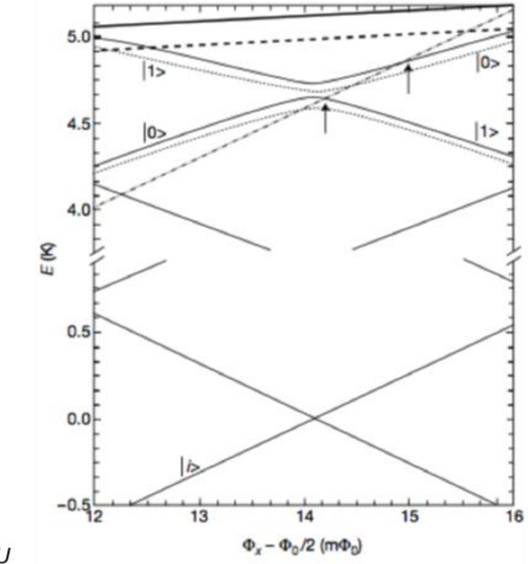


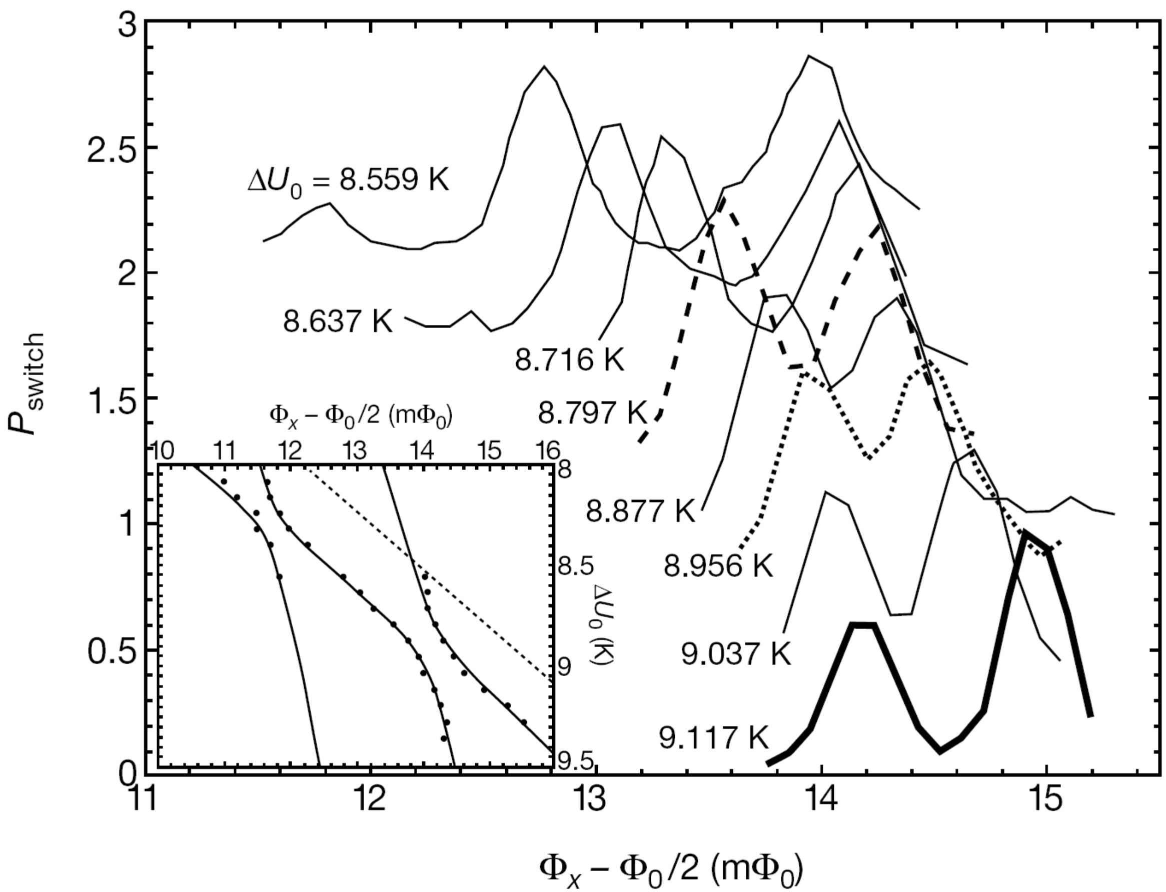


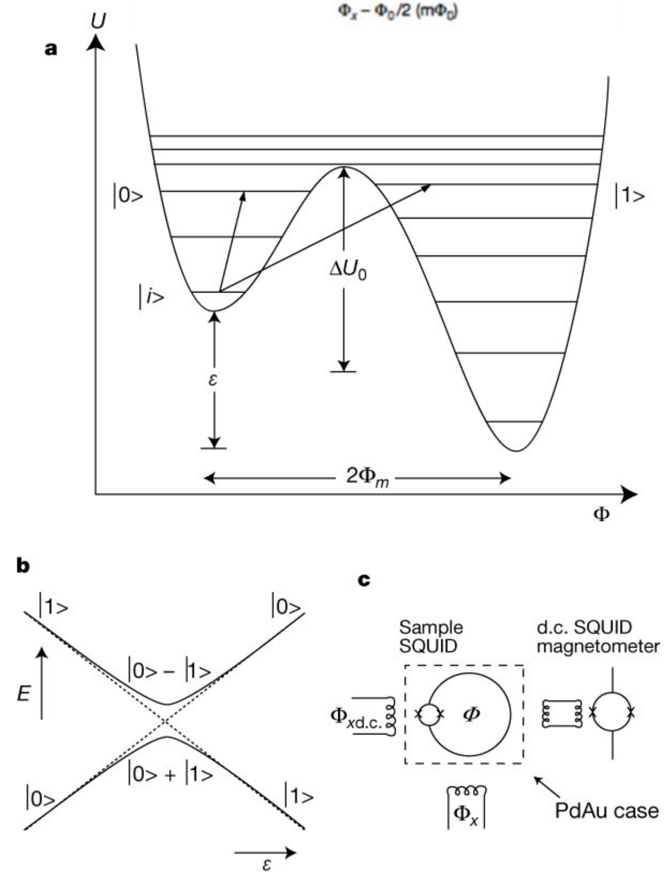
Clarke J. et al
Science 239 (992)
1988

Friedman J. R. et al
Nature 406 (43)
2000

SQUIDs

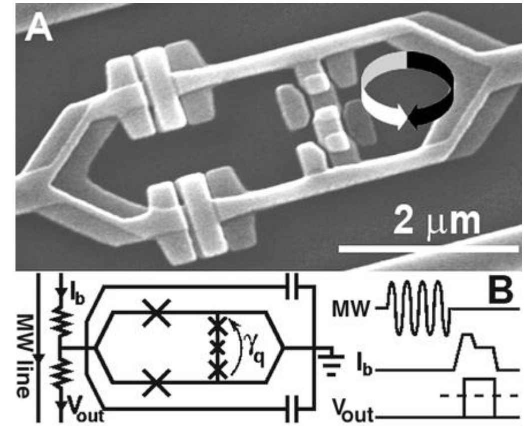


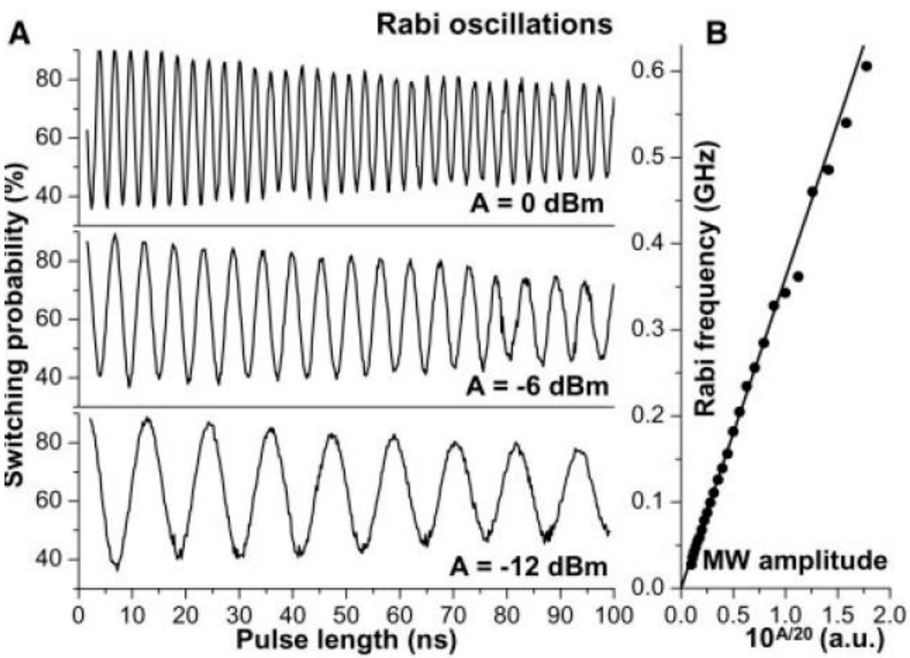


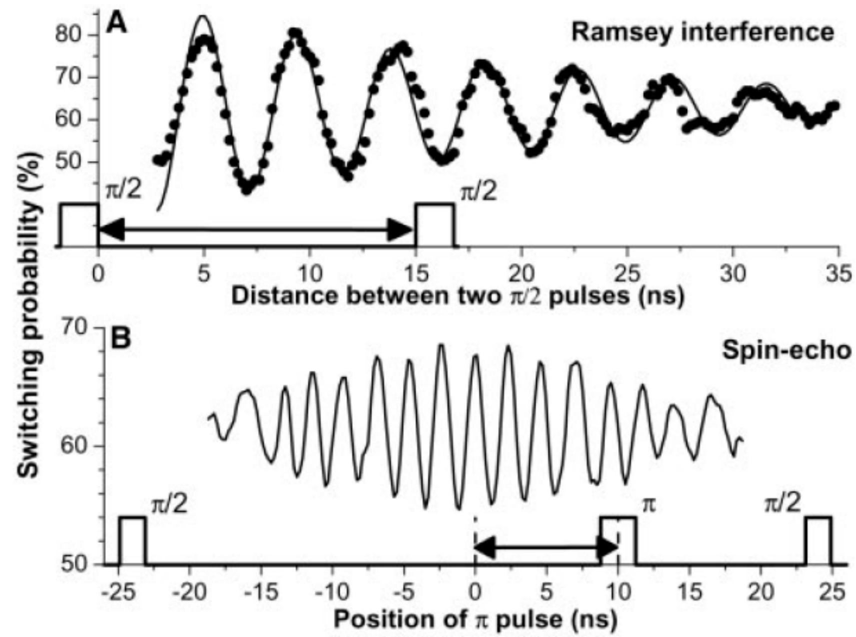


Chiorescu I. et al. Science 299, 1869, 2003

SQUIDs

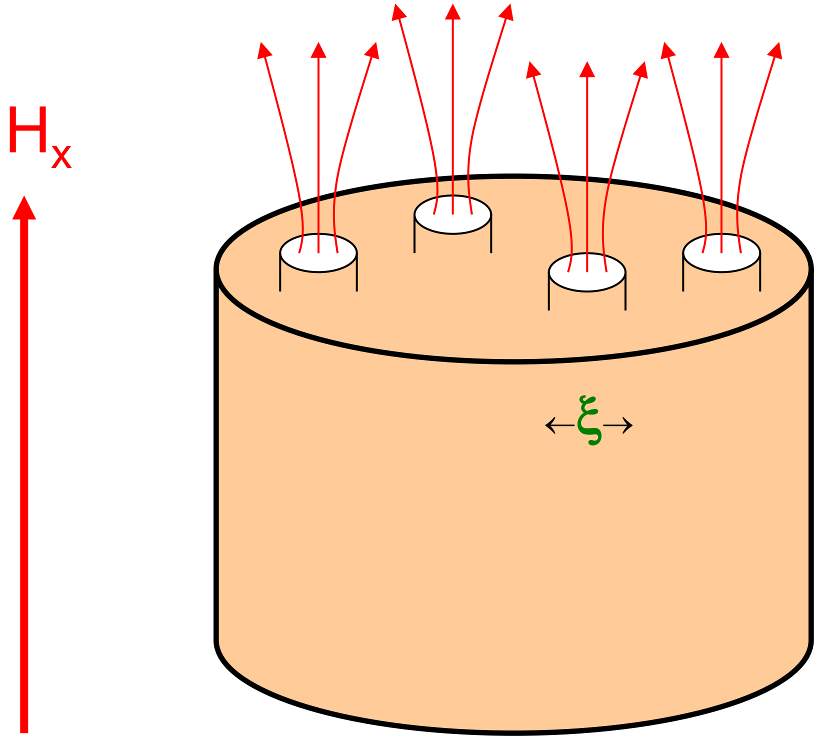


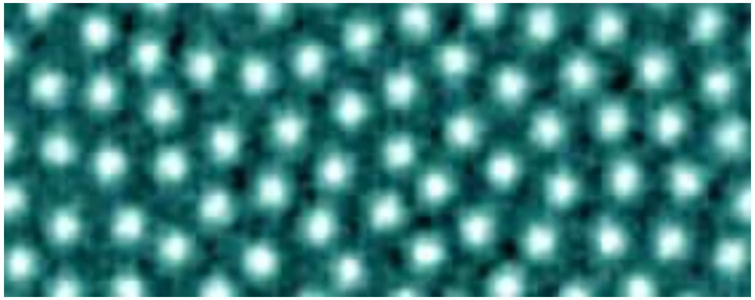




Vortices in superconductors

Type II superconductors





High T_c superconductors

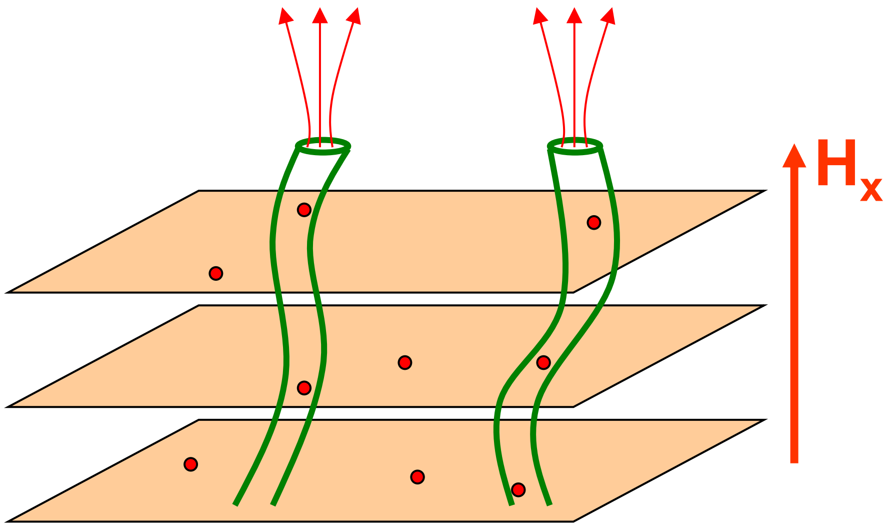
These are type II

$\lambda \gg \xi$ (λ is the penetration depth and ξ is the correlation length)

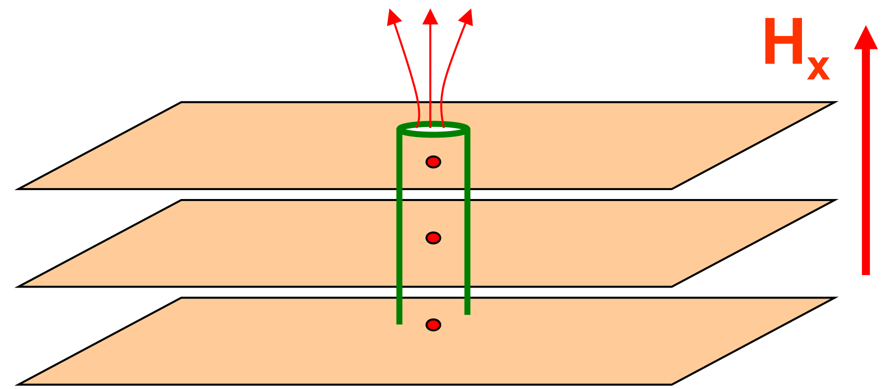
Quantum fluctuations are important!!

Vortex motion generates dissipation!

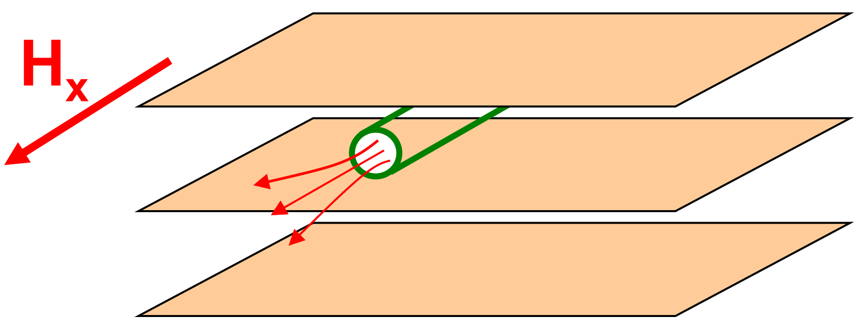
Vortices pinned by impurities



Pinning by columnar defects



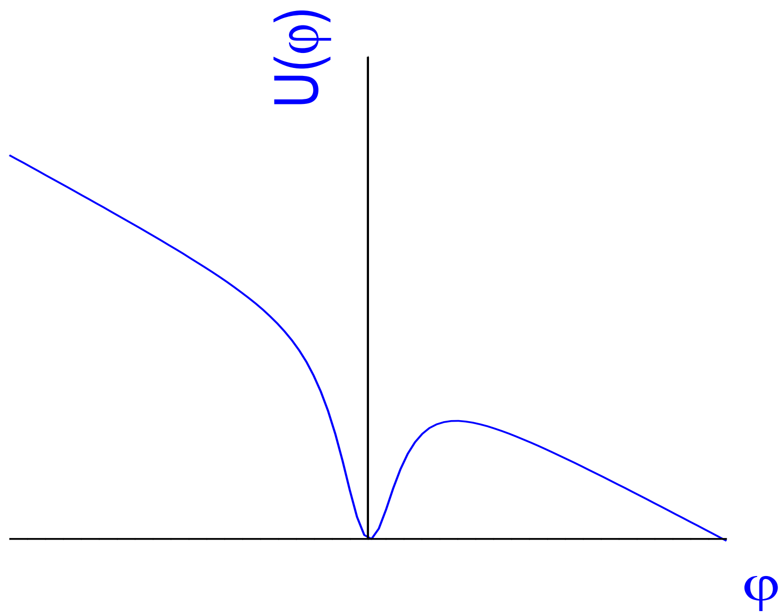
Intrinsically pinned vortices

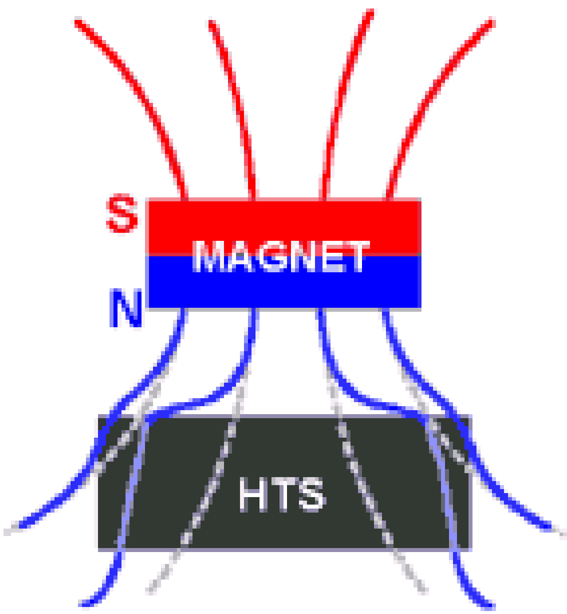


The dynamics of the transverse displacement of vortices can be described by equations like

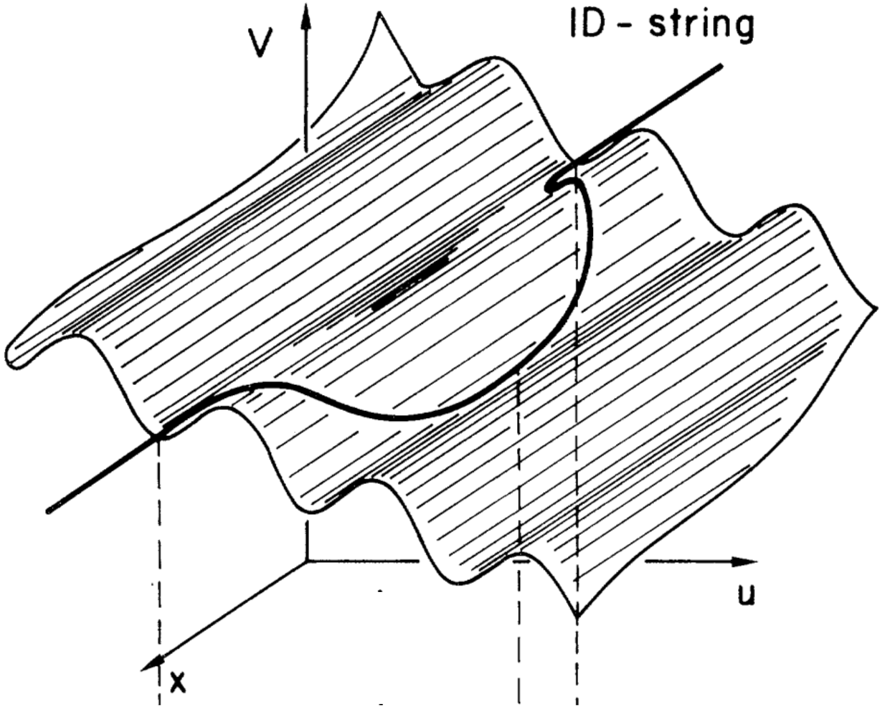
$$\frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} + \eta \frac{\partial \varphi}{\partial t} - \nabla^2 \varphi = -U'(\varphi)$$

where the potential U is of the form





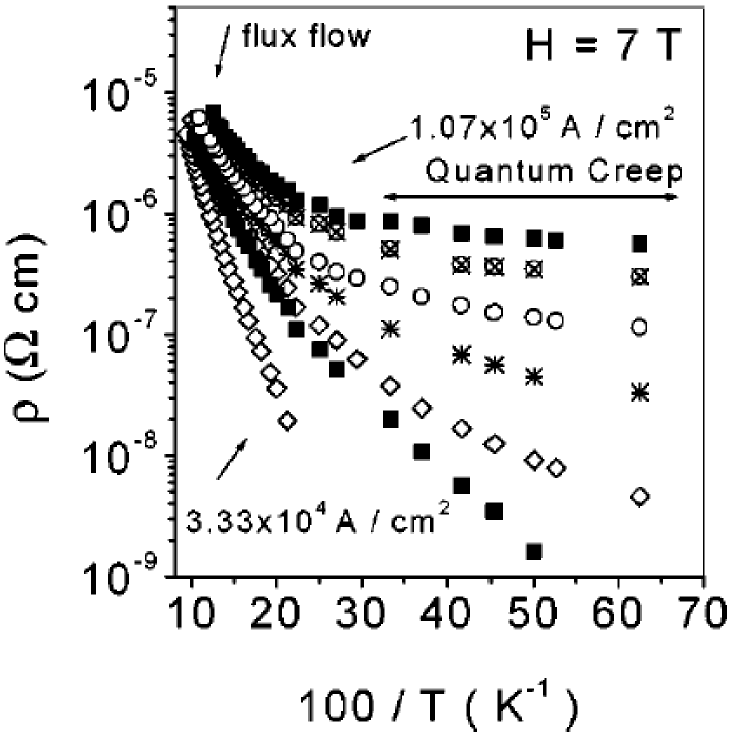
Equivalent mechanical problem

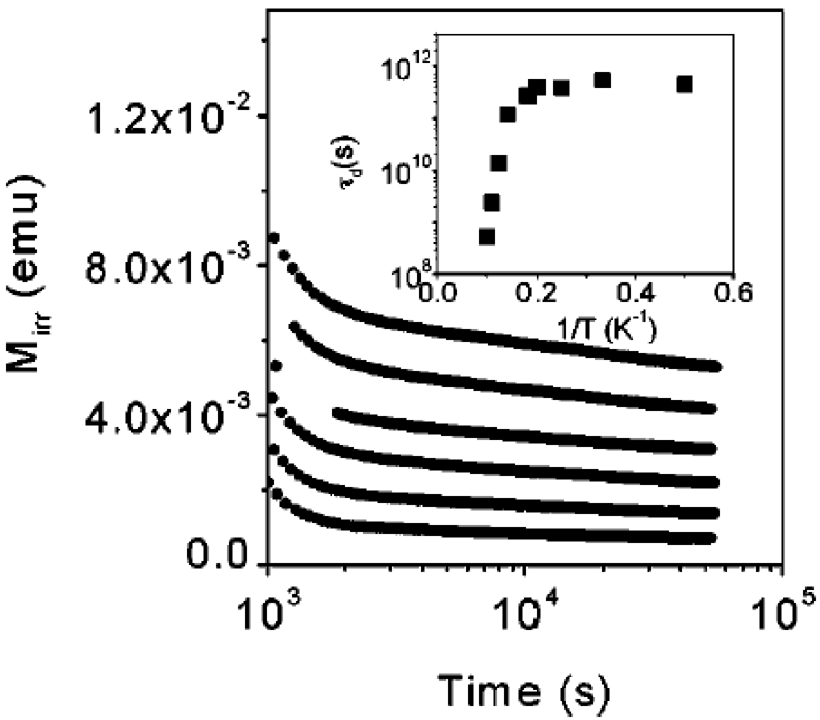


But...dissipation tends to inhibit quantum effects !

Experimental realizations

$Y\,Ba_2\,Cu_3\,O_{6.4}$ Thin films



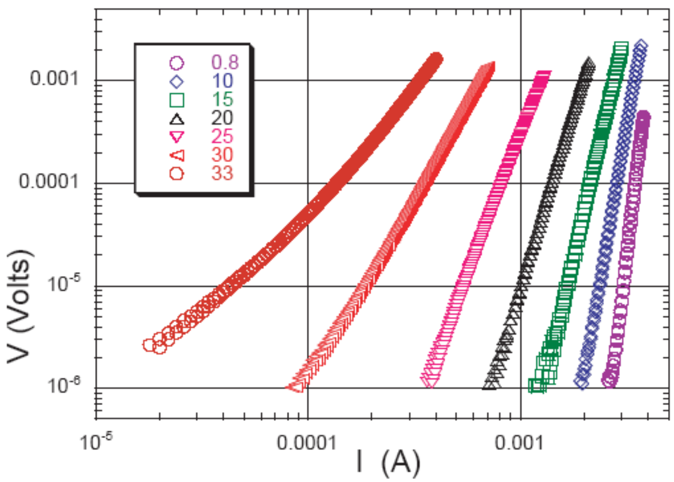


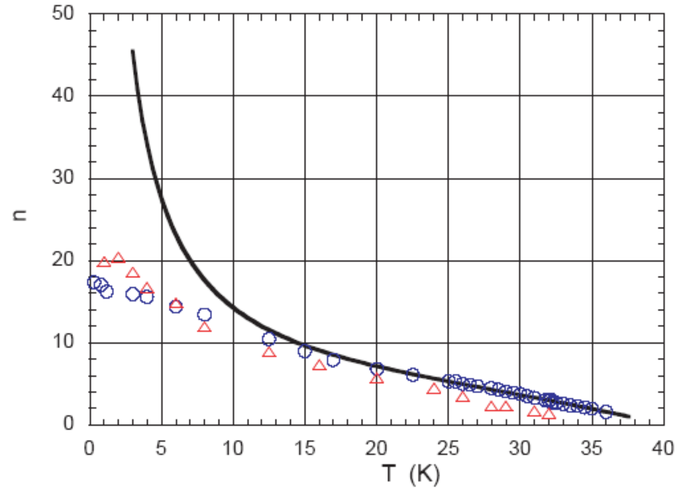
Sefrioui Z. et al. PRB 63, 054509, 2001

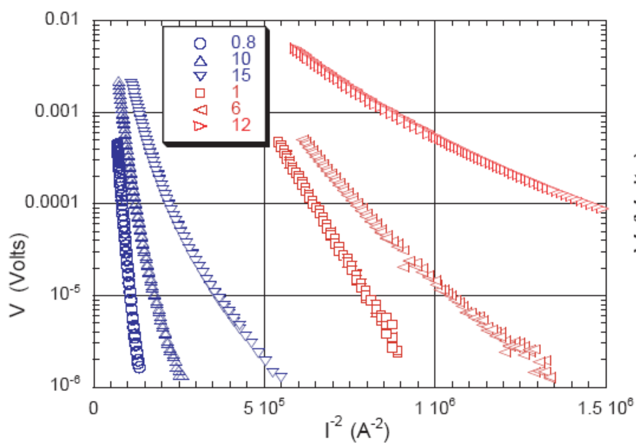
Ultra-thin current-carrying superconducting bridges

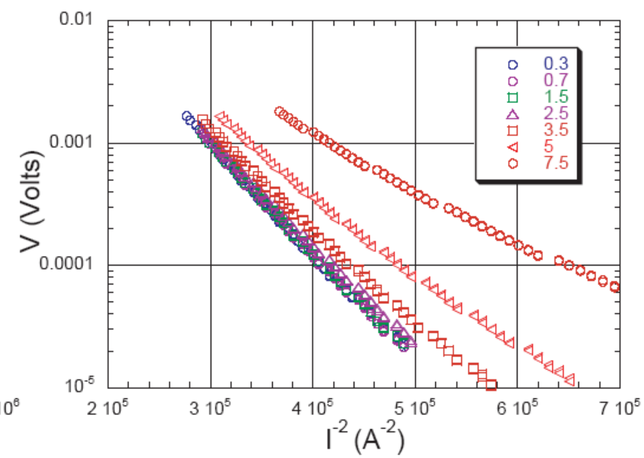
$V \propto I^n$ $\quad$ $\ln V \propto I^{-2}$

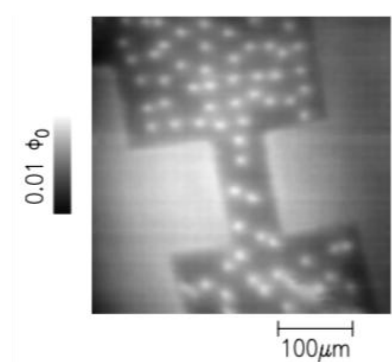
$$n = \frac{\phi_0^2}{8\pi^2 \Lambda(T) kT} + 1$$











Tafuri F. et al. EPL 73, 948, 2006

B. Superconducting qubits

Fundamental requirements for good qubits (DiVincenzo)

i) well-defined two state systems
ii) accuracy in preparing the initial state
iii) long phase coherence ; order of 10^4 coherent operations
iv) controllable effective fields
v) quantum measurement to read out the quantum information

Model hamiltonian $H = H_{qb} + H_{meas} + H_{env}$

$$H_{qb} = -\frac{1}{2}\sum_{i=1}^{N}\mathbf{B}^{(i)}(t)\cdot\vec{\sigma}^{(i)} + \sum_{i\neq j}\sum_{a,b} J_{ij}^{ab}(t)\sigma_a^{(i)}\sigma_b^{(j)}$$

$$a, b = x, y, z$$

We consider only 1 qubit coupled to its environment.

Flux qubit

This is the model we introduced before study quantum coherent tunnelling of the flux

$$H_{fqb} = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x$$

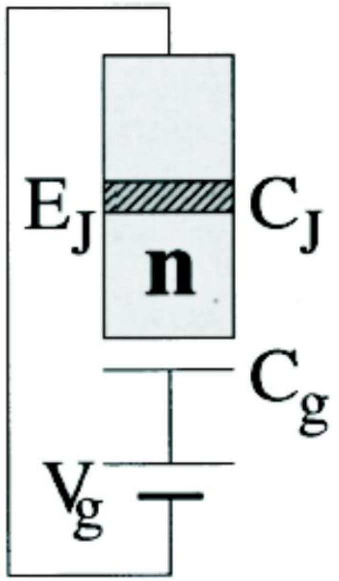
When the external flux bias is about half flux quantum

$$B_z(\phi_x) = 4\pi\sqrt{6(\beta_L - 1)}E_J\left(\frac{\phi_x}{\phi_0} - \frac{1}{2}\right) \qquad \beta_L \equiv 2\pi L i_0/\phi_0$$

This represents the bistable potential at half flux quantum.

We have addressed this problem before.

Charge qubit

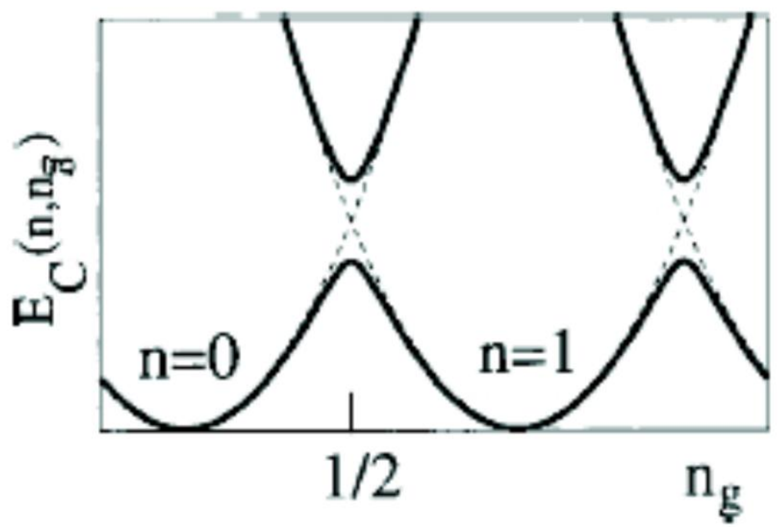


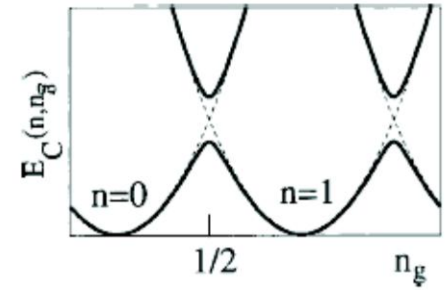
Charging energy $E_C = \frac{e^2}{2(C_J+C_g)}$

Cooper pair box Hamiltonian

$$H_{cqb} = 4E_C(n-n_g)^2 - E_J \cos\varphi$$

Canonical momentum $n = -i\hbar \frac{\mathrm{d}}{d(\hbar\varphi)}$





if $n_g = \frac{1}{2}$ our hamiltonian becomes

$$H_{cqb} = \sum_n [4E_C(n - n_g)^2 |n\rangle\langle n| - \frac{1}{2}E_J(|n\rangle\langle n+1| + |n+1\rangle\langle n|)]$$

Within the two-dimensional subspace

$$|\uparrow\rangle \equiv |n=0\rangle \text{ and } |\downarrow\rangle \equiv |n=1\rangle$$

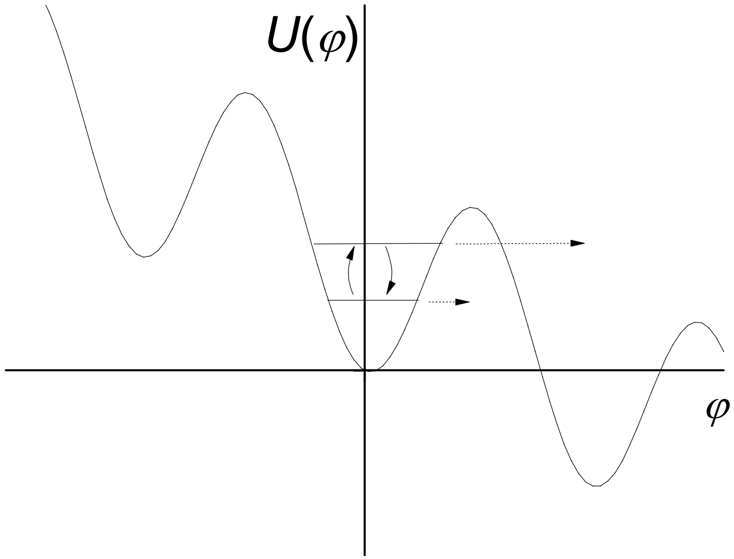
New two-state system hamiltonian $H_{cqb} = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x$

$$B_z = 4E_C(1 - 2n_g) \qquad \text{and} \qquad B_x = E_J$$

Transverse field can be tuned with a new circuit design

$$B_x = E_J(\phi_x) = 2E_J^0 \cos\left(\pi\frac{\phi_x}{\phi_0}\right)$$

Phase qubit and transmons



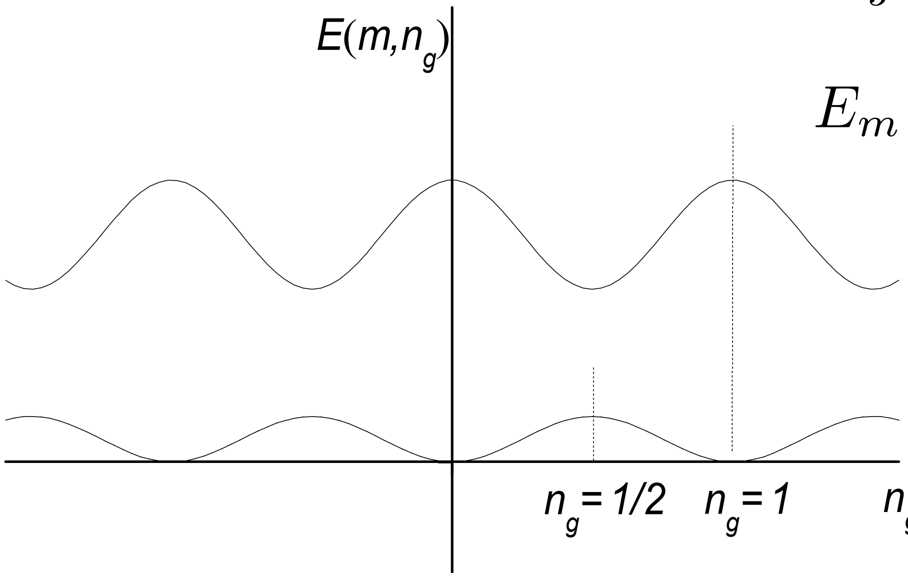
J. M. Martinis. S. Nam,
and J. Aumentado,
Phys. Rev. Lett. 89, 117901, (2002)

$$I(t) = I_{\rm dc} + \delta I_{\rm dc}(t)$$
$$+I_{\mu\rm wc}(t)\cos\omega_{10}t + I_{\mu\rm ws}(t)\sin\omega_{10}t$$

$$\delta I_{\rm dc}(t), I_{\mu\rm wc}(t),\ \text{and}\ I_{\mu\rm ws}(t)$$
$$\text{slow in comparison with } 2\pi/(\omega_{10}-\omega_{21})$$

$$H(t) = \frac{\sigma_z}{2}\,\delta I_{\rm dc}(t)\,\frac{\partial E_{10}}{\partial I_{\rm dc}} + \frac{\sigma_x}{2}\sqrt{\frac{\hbar}{2\omega_{10}C}}\,I_{\mu\rm wc}(t) + \frac{\sigma_y}{2}\sqrt{\frac{\hbar}{2\omega_{10}C}}\,I_{\mu\rm ws}(t)$$

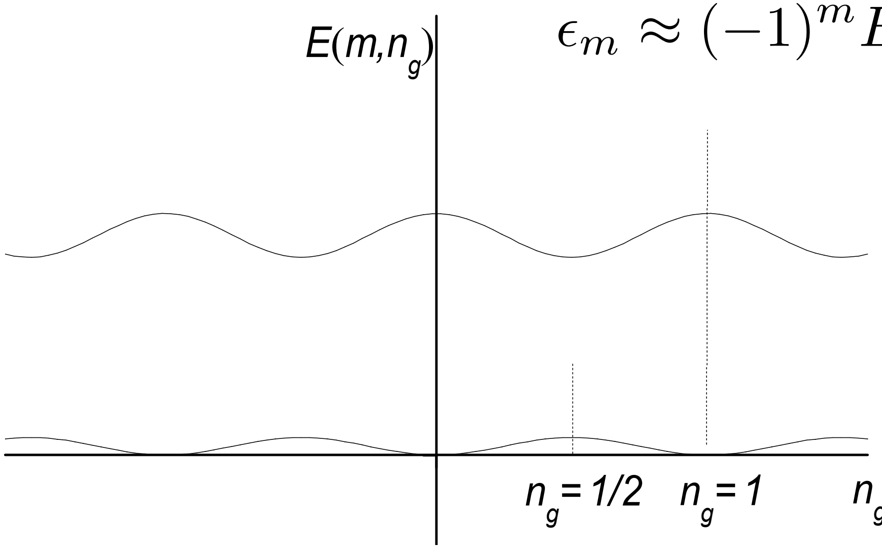
$$E_J \gg E_C$$



$$E_m(n_g) = E_m(n_g = 1/4) - \frac{\epsilon_m}{2}\cos(2\pi n_g)$$

$$\epsilon_m \equiv E_m(n_g = 1/2) - E_m(n_g = 0)$$

$$\epsilon_m \approx (-1)^m E_C \frac{2^{4m+5}}{m!}\sqrt{\frac{2}{\pi}}\left(\frac{E_J}{2E_C}\right)^{\frac{2m+3}{4}} \exp -\sqrt{\frac{8E_J}{E_C}}$$



$$E_m(n_g = 0) \approx -E_J + \sqrt{8E_J E_C}\left(m + \frac{1}{2}\right) - \frac{E_C}{12}(6m^2 + 6m + 3)$$

Decoherence

System dominated regime $\hbar\Delta \gg \alpha k_B T$

$\lambda \equiv \tan^{-1}(B_x/B_z)$

$\Delta = \sqrt{B_x^2 + B_z^2}$

$$\tau_{rel}^{-1} = \pi\,\alpha\sin^2\lambda\;\Delta\coth\frac{\hbar\Delta}{2k_BT}$$

$$\tau_{\varphi}^{-1} = \frac{1}{2}\,\tau_{rel}^{-1} + \pi\,\alpha\cos^2\lambda\;\frac{2k_BT}{\hbar}$$

In general $$\mathcal{H} = \mathcal{H}_{qb} + \sum_i \boldsymbol{\sigma}\cdot\mathbf{n}_i\left(\sum_a C_a^{(i)} q_a^{(i)}\right) + \sum_i \mathcal{H}_{env}^{(i)}$$

$\hbar\Delta E \gg \sum_i \alpha_i k_B T$

$\cos\lambda_i \equiv \mathbf{B}\cdot\mathbf{n}_i/|\mathbf{B}|$

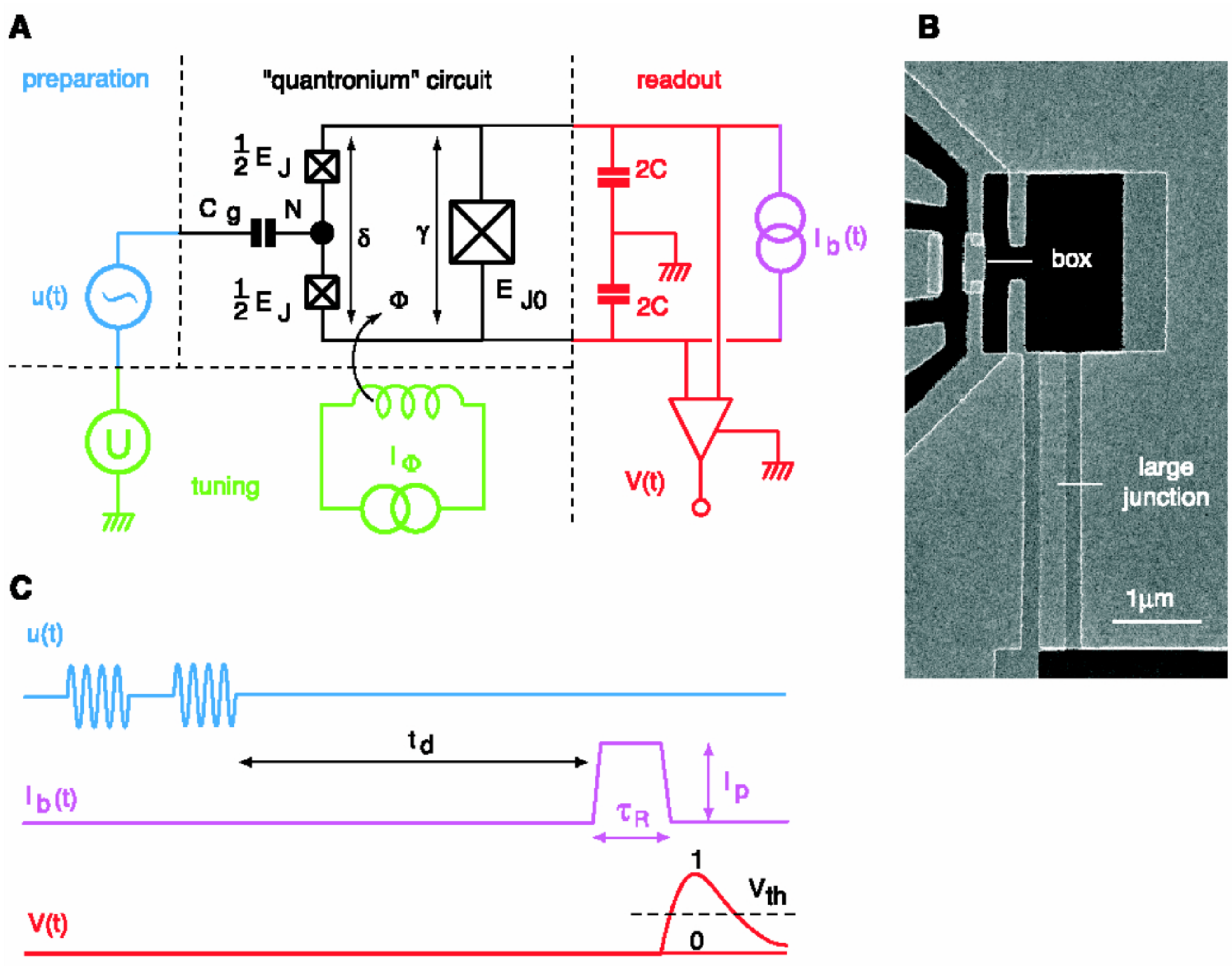
$$\tau_{rel}^{-1} = \sum_i \pi\,\alpha_i\sin^2\lambda_i\;\Delta\coth\frac{\hbar\Delta}{2k_BT}$$

$$\tau_{\varphi}^{-1} = \frac{1}{2}\,\tau_{rel}^{-1} + \sum_i \pi\,\alpha_i\cos^2\lambda_i\;\frac{2k_BT}{\hbar}$$

Experimental results

Cooper Pair Box

D. Vion *et al* Science 296, 886 (2002)



Rabi oscillation and Ramsey fringe experiments to measure the Switching probability

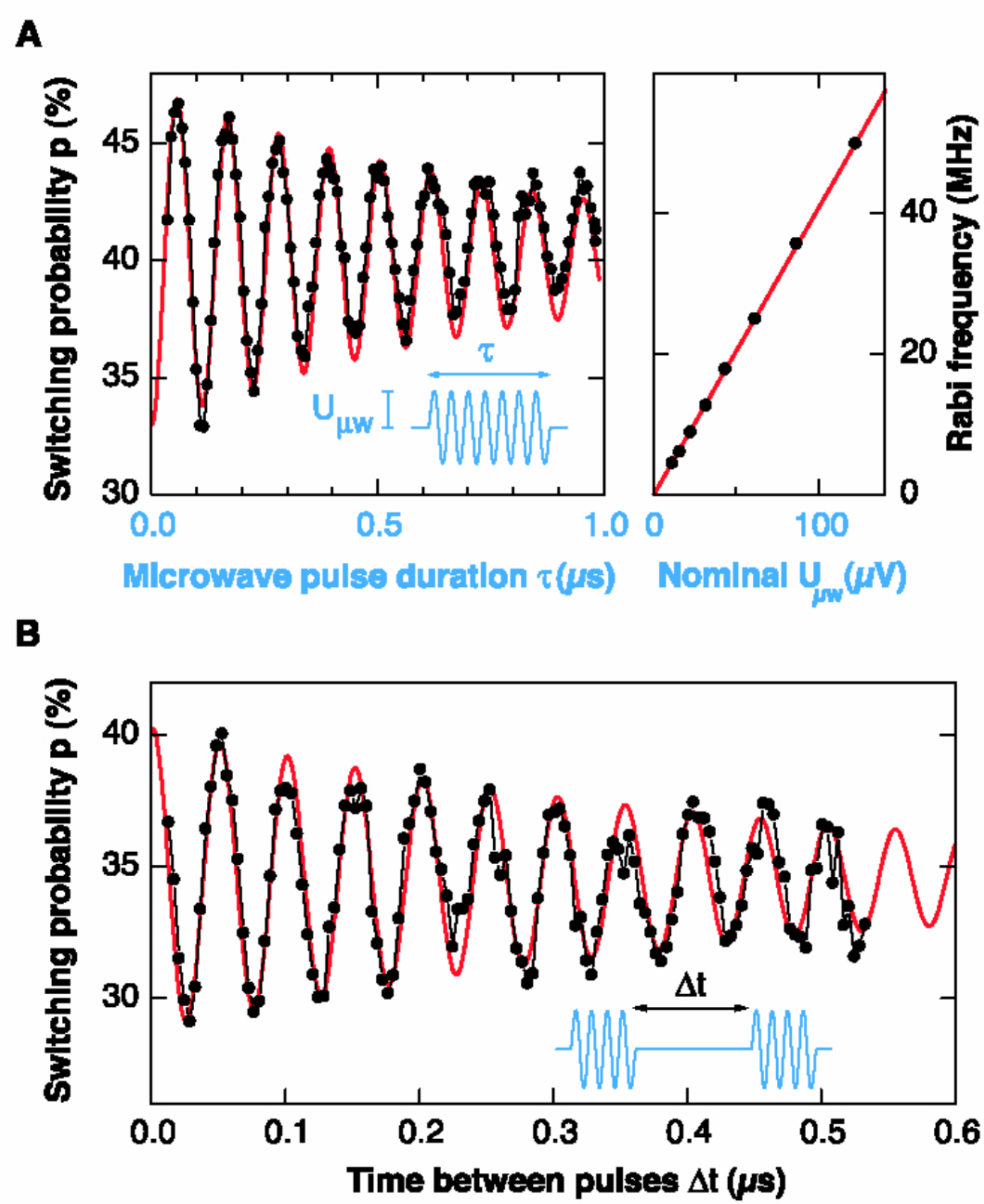
$$E_J = 0.865 k_B K$$

$$E_J/E_C = 1.27$$

$$T = 15mK$$

$$\tau_\varphi \approx 0.50\mu s$$

$$1/\nu_{01} \approx 60ps$$

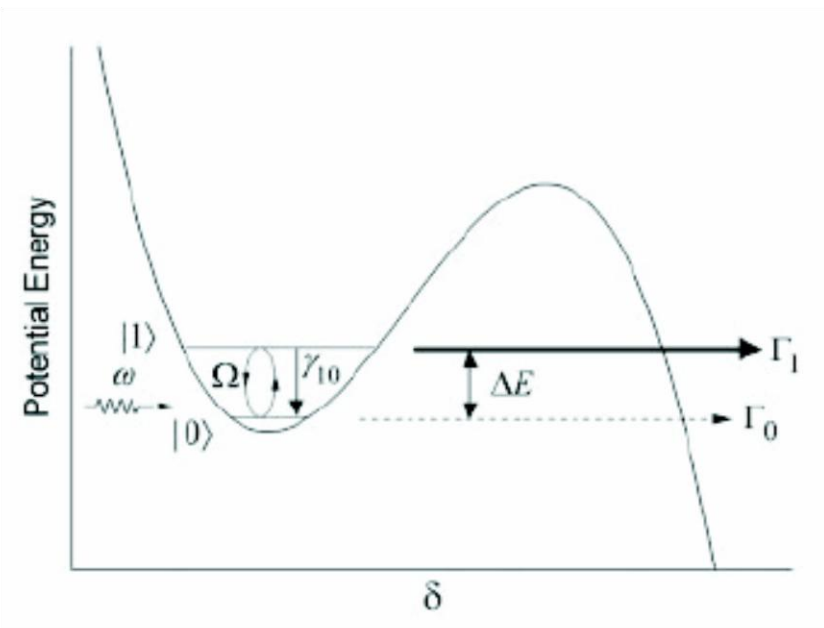


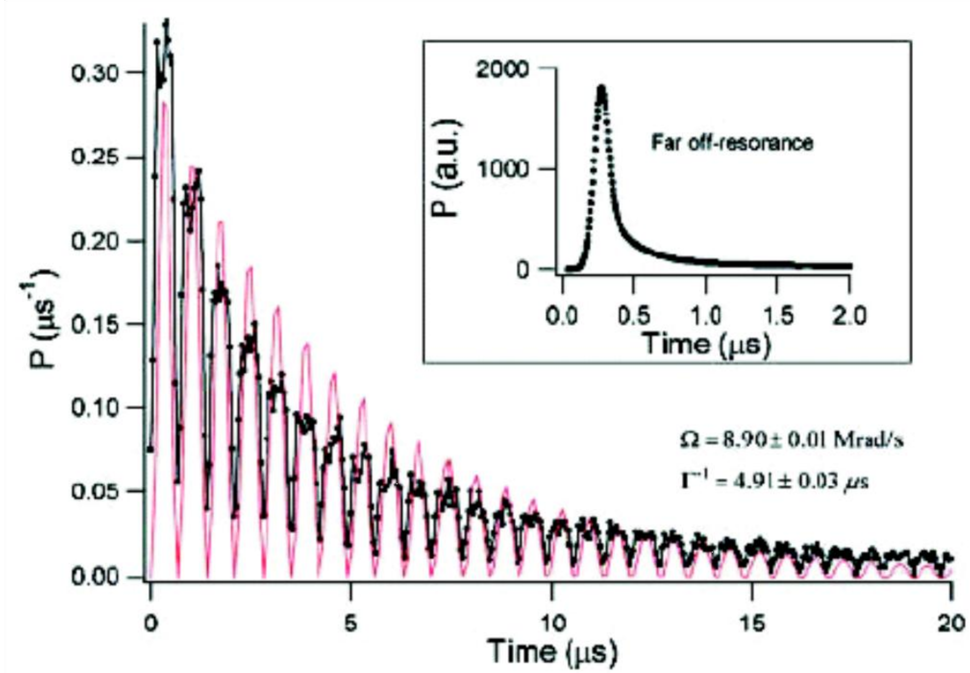
Excited CBJJ

Y. Yu *et al* Science 296, 889 (2002)

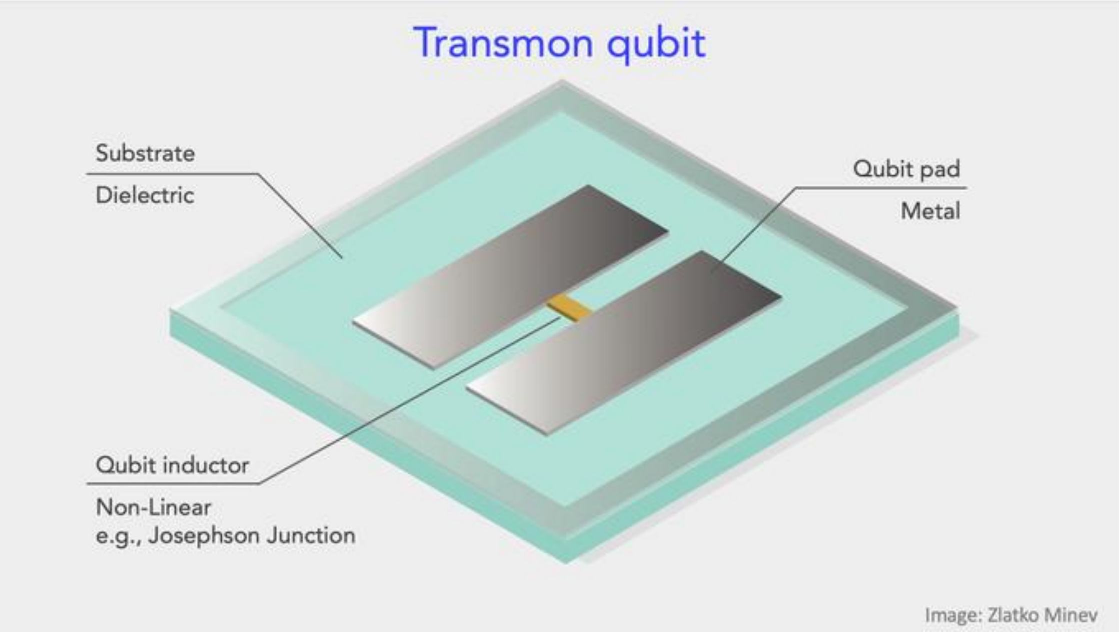
Measurement of the decay rate of the excited state of a CBJJ via Rabi oscillations

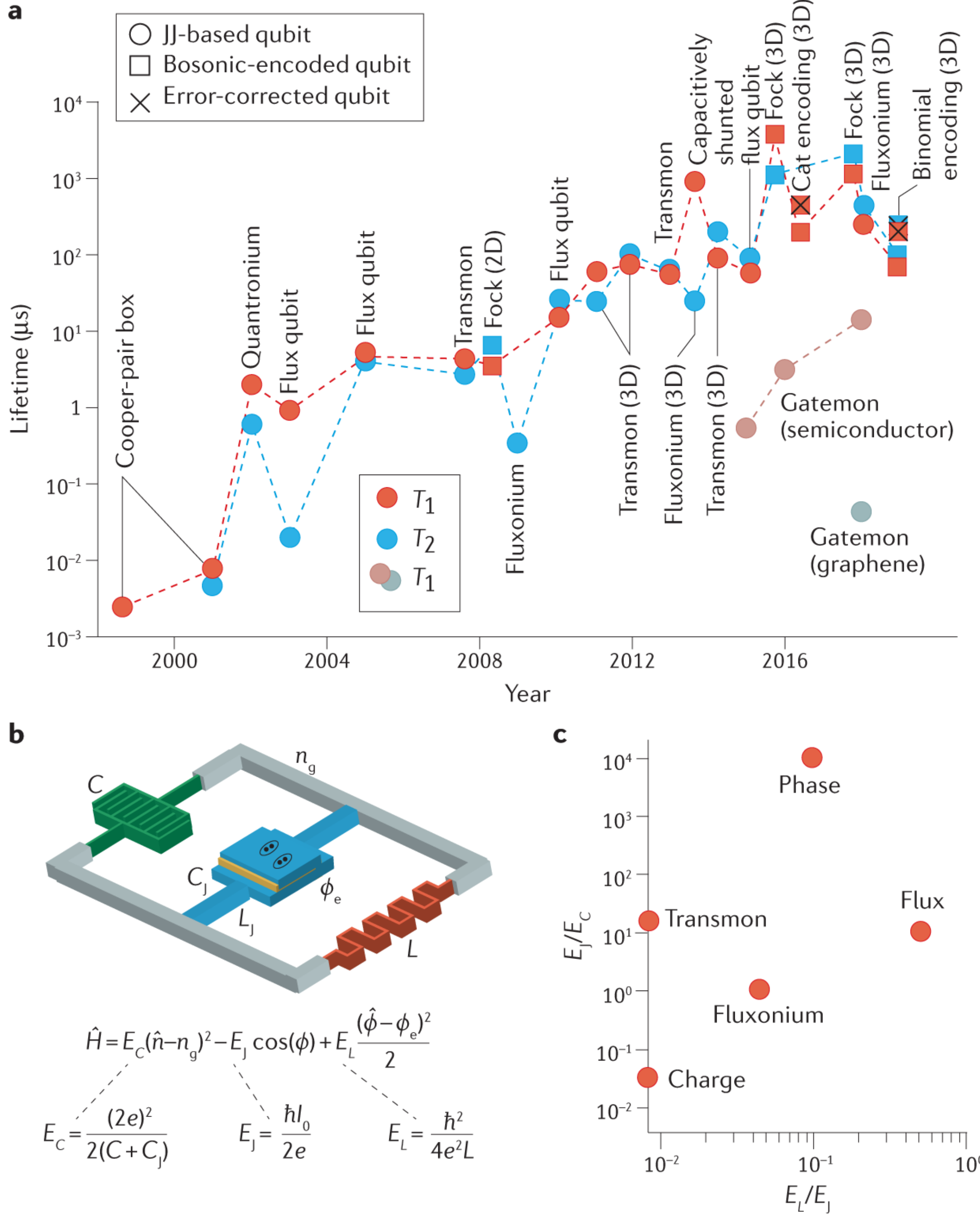
$T = 8mK$ $\tau_\varphi \gtrsim 4.9 \mu s$





If $E_C \leq E_J$ and the CBJJ is subjected to a "gate voltage" we have a transmon





a
JJ-based qubit
Bosonic-encoded qubit
Error-corrected qubit
Lifetime (μs)
10^4
10^3
10^2
10^1
1
10^{-1}
10^{-2}
10^{-3}
Cooper-pair box
Quantronium
Flux qubit
Flux qubit
Transmon
Fock (2D)
Fluxonium
Flux qubit
Transmon (3D)
Transmon
Fluxonium (3D)
Transmon (3D)
Capacitively shunted flux qubit
Fock (3D)
Cat encoding (3D)
Fock (3D)
Fluxonium (3D)
Binomial encoding (3D)
Gatemon (semiconductor)
Gatemon (graphene)
T_1
T_2
T_1
2000
2004
2008
2012
2016
Year
b
C
n_g
C_J
ϕ_e
L_J
L
$\hat{H} = E_C(\hat{n}-n_g)^2 - E_J\cos(\phi) + E_L\frac{(\hat{\phi}-\phi_e)^2}{2}$
$E_C = \frac{(2e)^2}{2(C+C_J)}$
$E_J = \frac{\hbar I_0}{2e}$
$E_L = \frac{\hbar^2}{4e^2L}$
c
E_J/E_C
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
Phase
Transmon
Flux
Fluxonium
Charge
10^{-2}
10^{-1}
10^0
E_L/E_J

Final comments

a) Importance of properly chosen superconducting systems to study quantum mechanics at system sizes larger than usual. Particular emphasis given to superconducting devices, but the physical phenomenon underlying this possibility is also present in bulk superconductors, BECs and in many other systems.

b) Those characteristics of the chosen systems, on the other hand, make them very sensitive to the coupling to the external world. Macroscopic quantum phenomenon and dissipative effects are, in principle, inseparable.

c) Semi – phenomenological approach to quantum dissipation has proved quite useful for treating problems as ordinary relaxation, decoherence, decay by quantum tunnelling or coherent tunnelling.

Final comments

d) Feynman path integral method provides us with a unified way to tackle all these different problems. It also allows us to deduce master equations for describing our systems under certain circumstances.In particular, these are valid either for high temperatures at any damping or very low damping at any temperature.

e) The collective coordinate method (collision model), very familiar to field theorists, can be very useful to study the dynamics of topological excitations in material media. Examples are, bright or dark solitons and vortices in BECs, skyrmions and domain walls in magnetic systems etc.

f) Experimental realizations so far; vortex dynamics in bulk and 2 – D superconductors, superconducting devices, magnetic particles, and nanoelectromechanical devices among others.

Final comments

g) Relevance for:

- Testing quantum mechanics at the macroscopic level. Foundations: quantum theory of measurement ?

- Improvement of the coherence time aiming at technological applications; quantum computing and quantum information. Qubits.

- Quantum engines and quantum thermodynamics.

- Other applications ommited either for the lack of time or ignorance.

Thank you !

Vortices

Fraction of electrons per unit length localized within the flux tube $2\pi\xi^2 N(E_F)\delta\epsilon$ where $\delta\epsilon \simeq \hbar v_F/\pi\xi$ and

$$N(E_F) = m_e k_F/2\pi^2\hbar^2$$

Change of the confined mass of electrons within the core

$$m_e\delta\epsilon/E_F$$

Linear density of mass of the vortex line is

$$m_l = \frac{2}{\pi^3} m_e k_F$$

This linear density can also be of other origins