Measurements beyond standard quantum limits and their applications

QUANTOP

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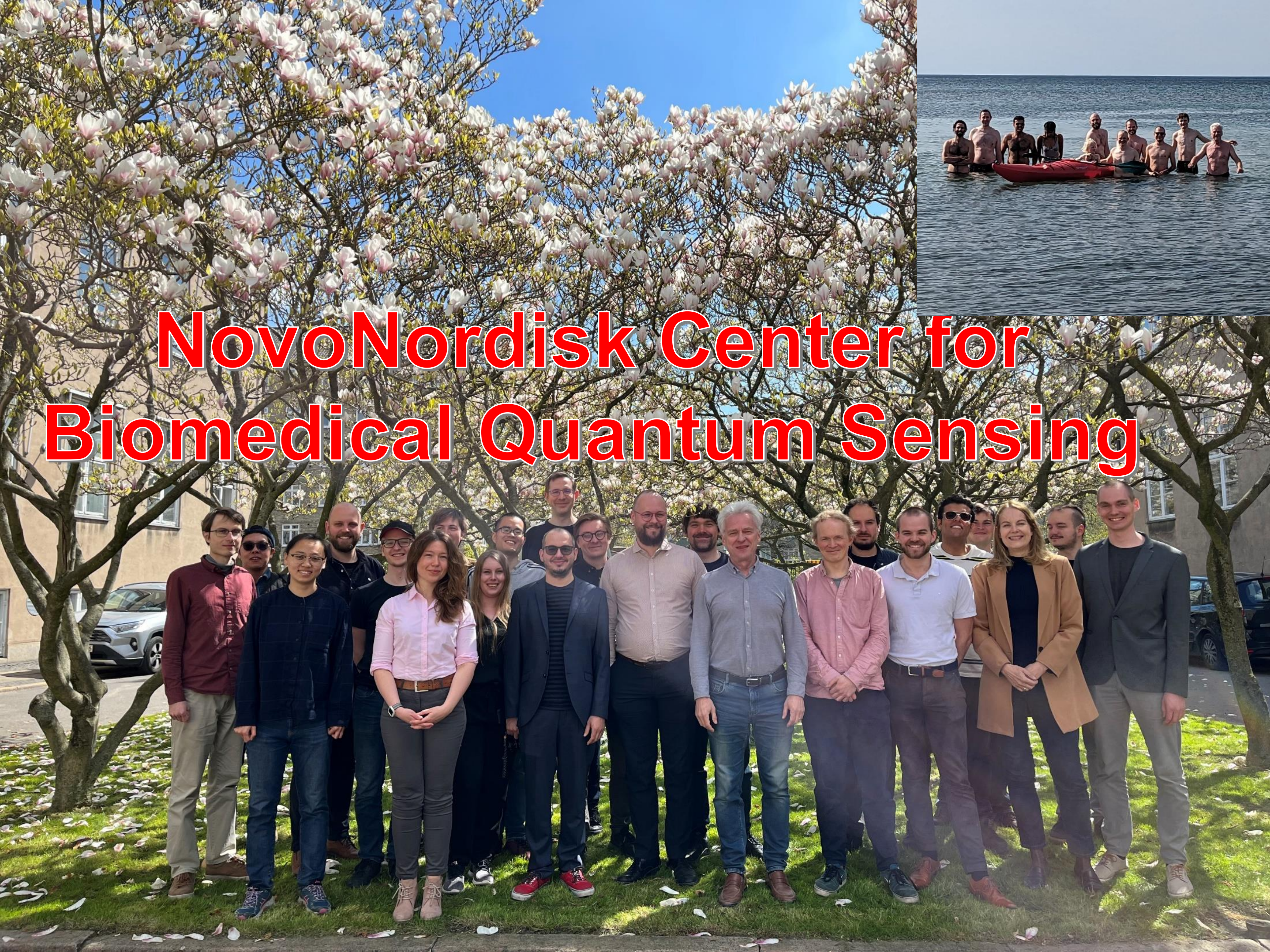
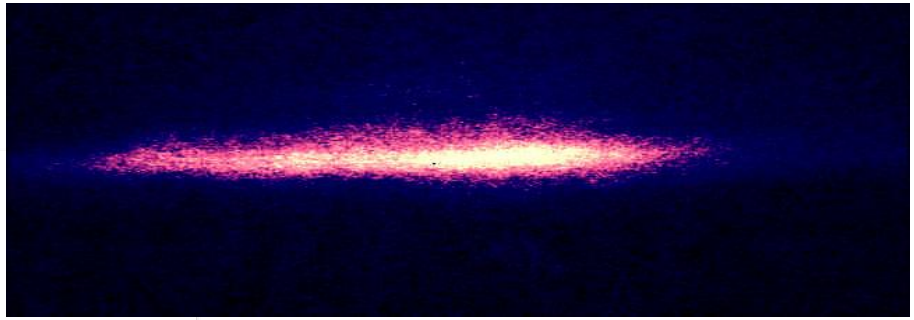
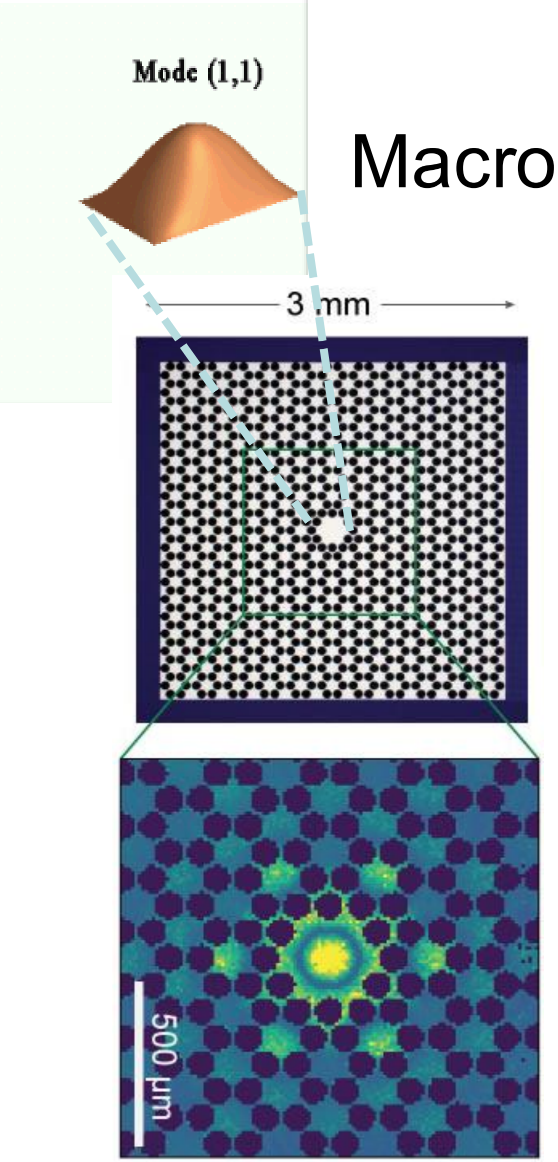
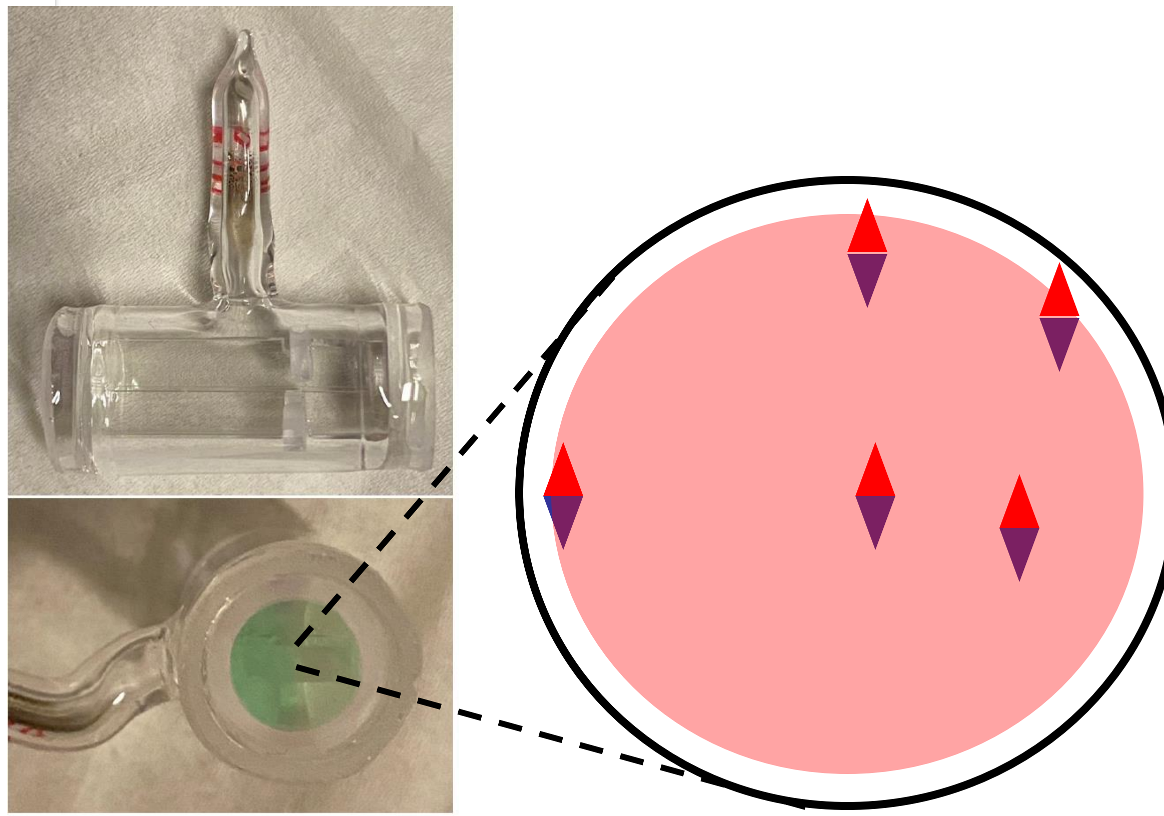




Macroscopic objects in quantum regime





Collective excitations and canonical variables for:

Light modes

Ensembles of polarized atoms (spins)

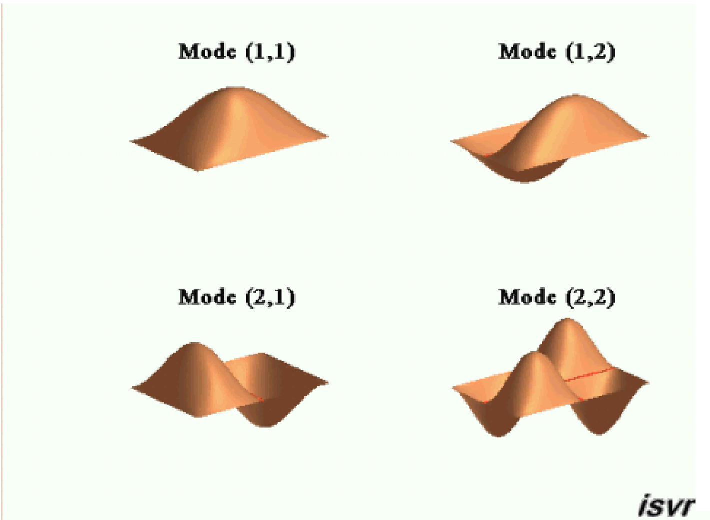
Solid state oscillators

K. Hammerer, A. Sørensen, E.P. Reviews of Modern Physics, April 2010
arXiv:0807.3358

Continuous variable quantum systems

$$\hat{X}_{lab}(t) = \hat{X}\sin(\omega t) + \hat{P}\cos(\omega t)$$

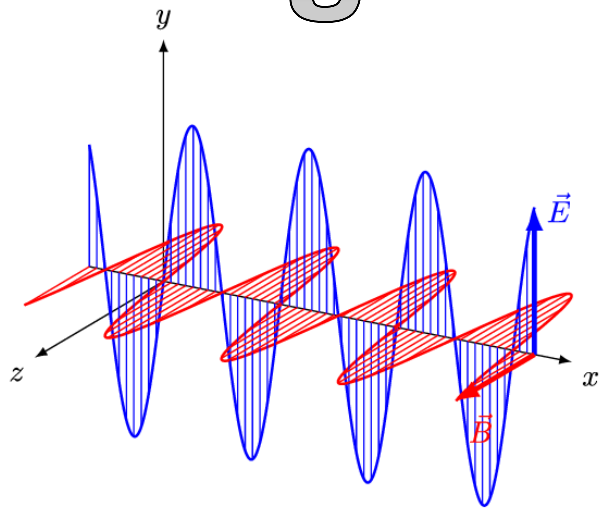
Mechanical



$\hat{X}$ - position

$\hat{P}$ - momentum

Light

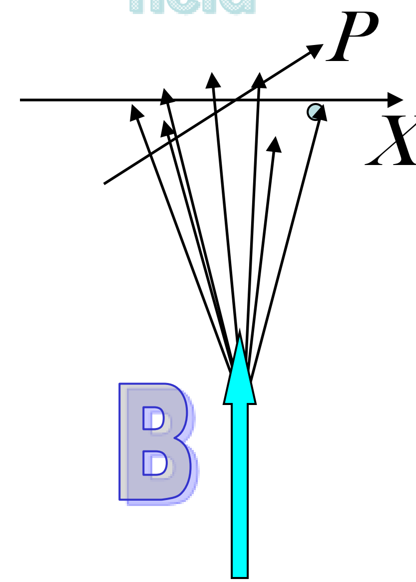


$\hat{X}$ - Phase quadrature

$\hat{P}$ - Amplitude quadrature

Spin

in magnetic field



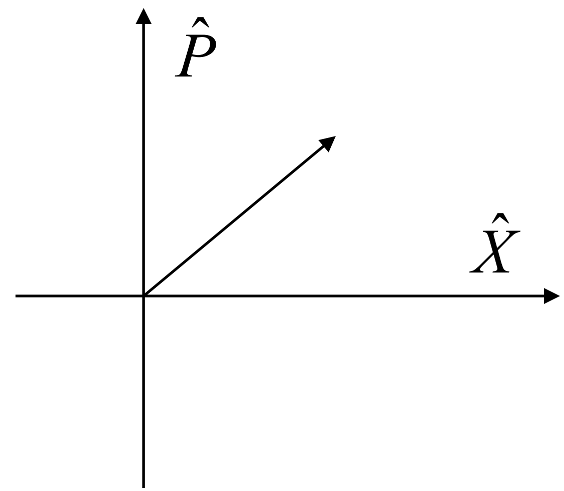
$\hat{X}$ $\hat{P}$ - spin projections

Canonical quantum variables for light:

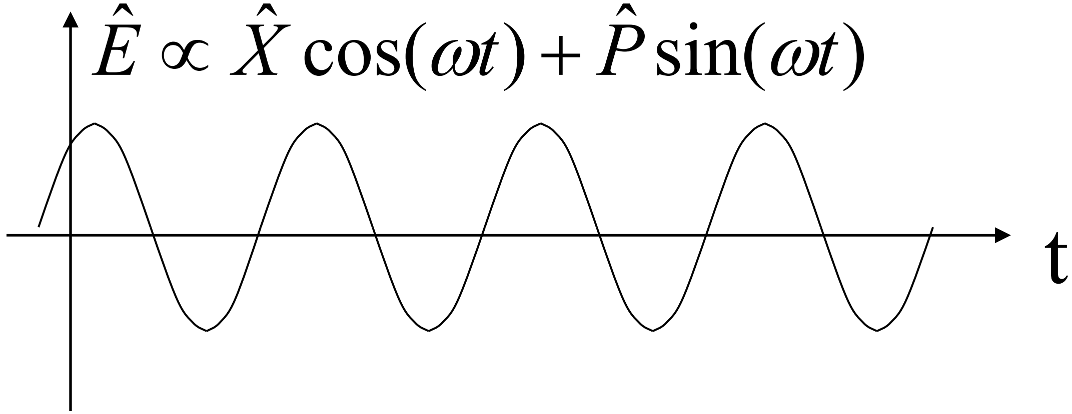
•E.-m. field is described by two noncommuting variables: *amplitude and phase quadratures, X and P*

$$\hat{X} = \frac{1}{\sqrt{2}}(\hat{a}^{+} + \hat{a}), \quad \hat{P} = \frac{i}{\sqrt{2}}(\hat{a}^{+} - \hat{a}) \qquad \left[\hat{X}, \hat{P}\right] = i$$

$$[\hat{a}, \hat{a}^{+}] = 1 \qquad \left[\hat{X}, \hat{P}\right] = \hat{X} \cdot \hat{P} - \hat{P} \cdot \hat{X} = i$$



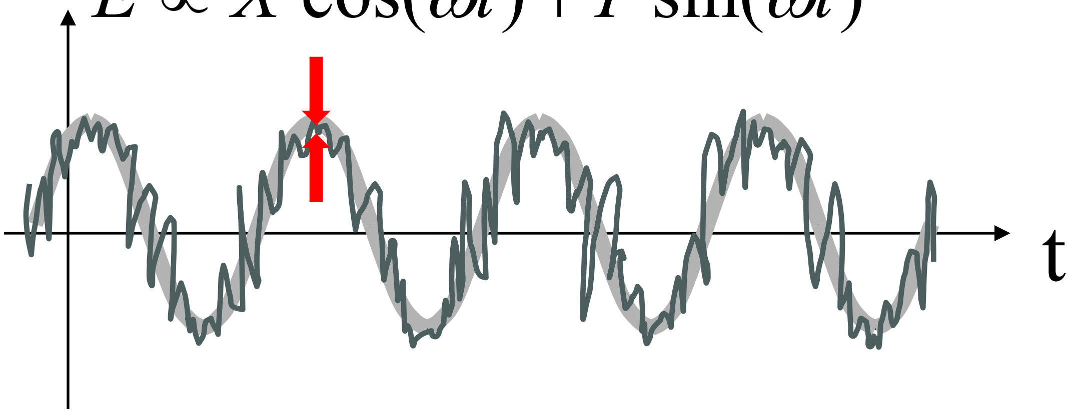
$$\hat{E} \propto \hat{X}\cos(\omega t) + \hat{P}\sin(\omega t)$$



Uncertainty relation for field operators. Coherent and Squeezed States

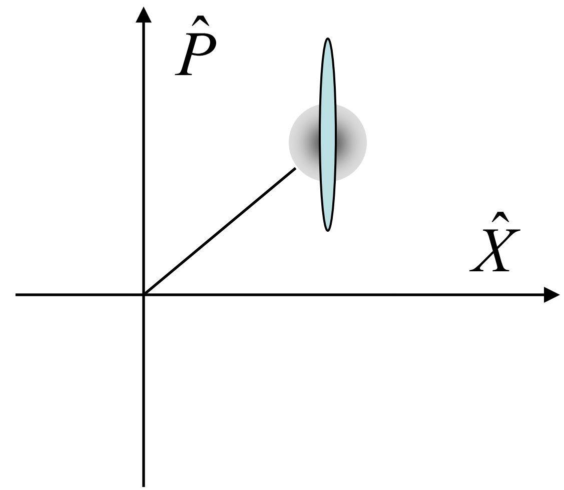
$$[\hat{X}, \hat{P}] = i \Longrightarrow \delta X \delta P \geq 1/2$$

$$\hat{E} \propto \hat{X}\cos(\omega t) + \hat{P}\sin(\omega t)$$



Coherent state (or vacuum)

$$\delta X^2 = \delta P^2 = 1/2$$



Squeezed state

$$\delta X^2 < 1/2$$

Two-mode squeezed (EPR)

$$\delta(X_1 - X_2)^2 + \delta(P_1 + P_2)^2 < 2$$

Polarization homodyning - measurement of X (or P)

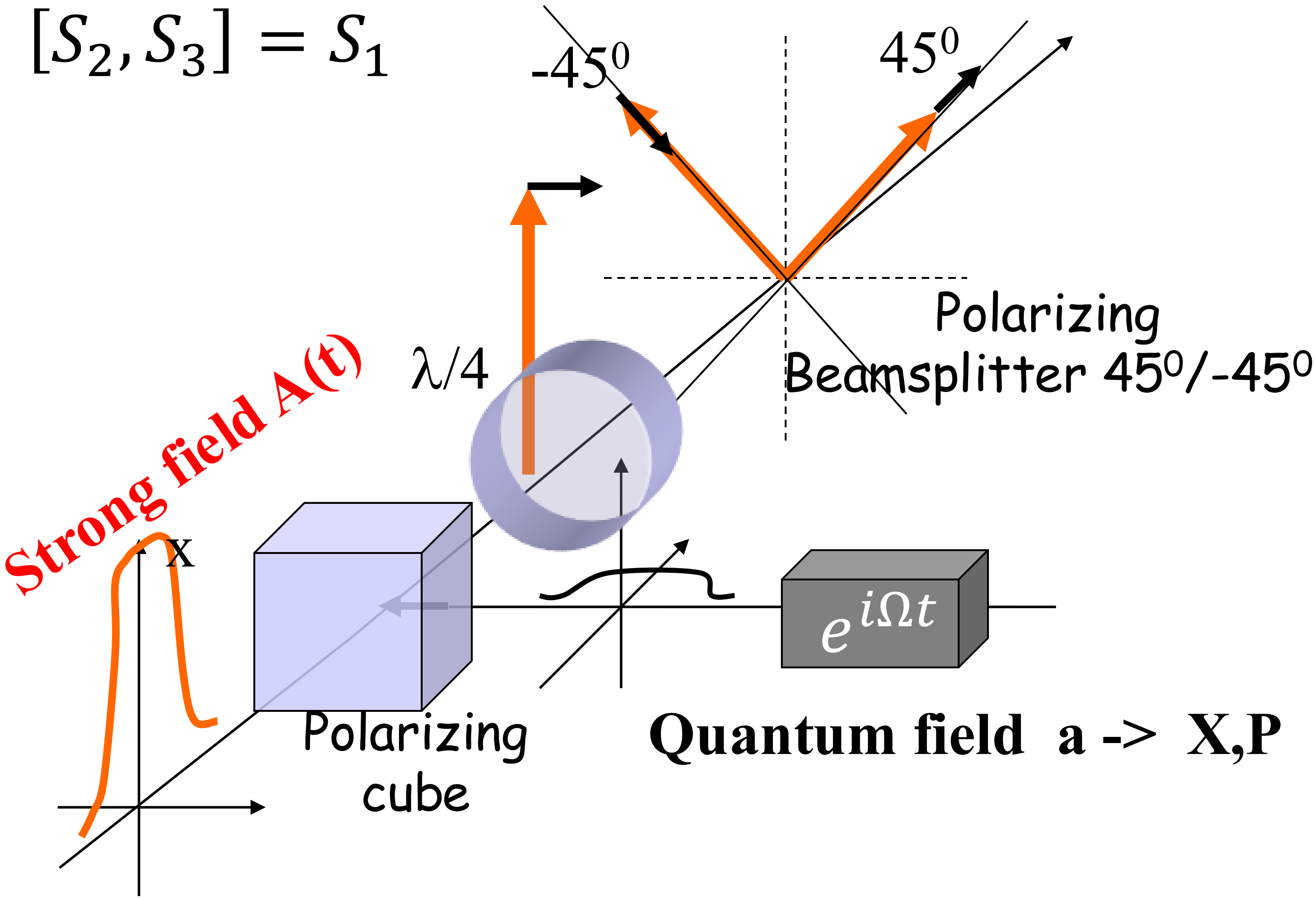
$$\hat{S}_2 = \tfrac{1}{4}[(A+\hat{a})^+(A+\hat{a}) - (A-\hat{a})^+(A-\hat{a})] =$$

$$\tfrac{1}{2}A(a^+ + a) = \tfrac{1}{\sqrt{2}}A\hat{X}cos\Omega t$$

Stokes operators

$$[S_2, S_3] = S_1$$

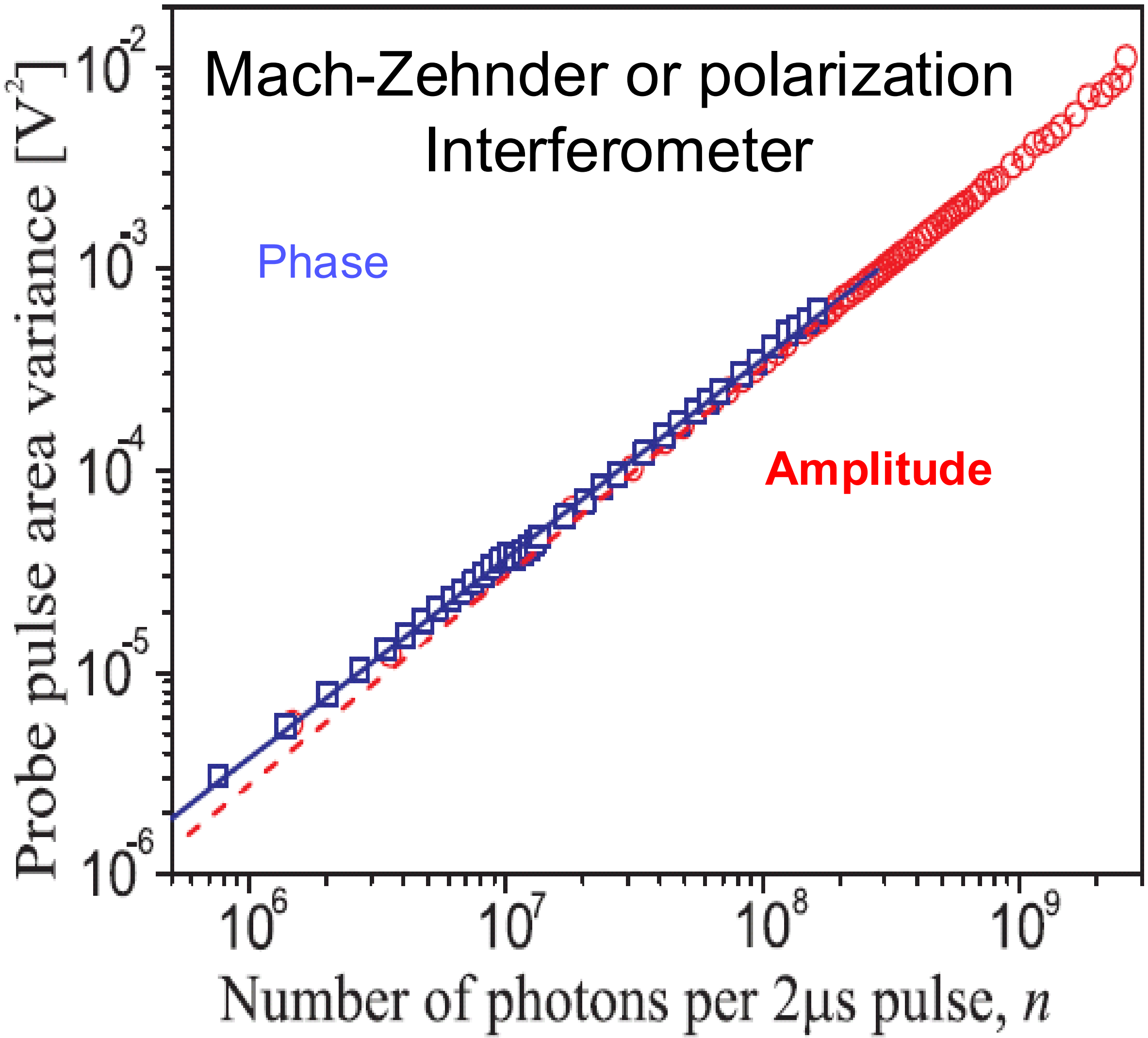
$$\hat{S}_3 = \tfrac{1}{\sqrt{2}}A\hat{P}$$



Benchmark I: quantum noise of Coherent State of Light – $VarX, P \propto N$

•Photodetectors with q.e.>99% and dark noise << shot noise of light

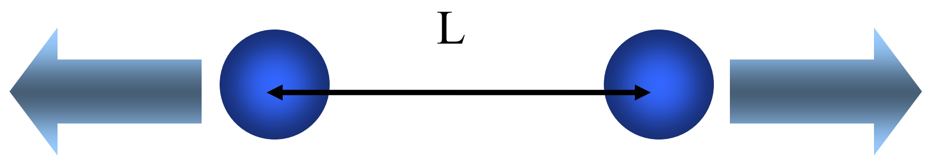
•Stabilization of phase and amplitude noise of light down to the SQL = shot noise



Einstein-Podolsky-Rosen (EPR) entanglement 1935

2 particles entangled
in position/momentum

$$\left[\hat{X}_1 - \hat{X}_2, \hat{P}_1 + \hat{P}_2\right] = 0$$



$\hat{X}_1, \hat{P}_1$ $\hat{X}_2, \hat{P}_2$

$$\hat{X}_1 - \hat{X}_2 = L \qquad \hat{P}_1 + \hat{P}_2 = 0$$

Minimal uncertainty state of both particles: $\delta X_{1,2}^2 = \delta P_{1,2}^2 = 1/2$

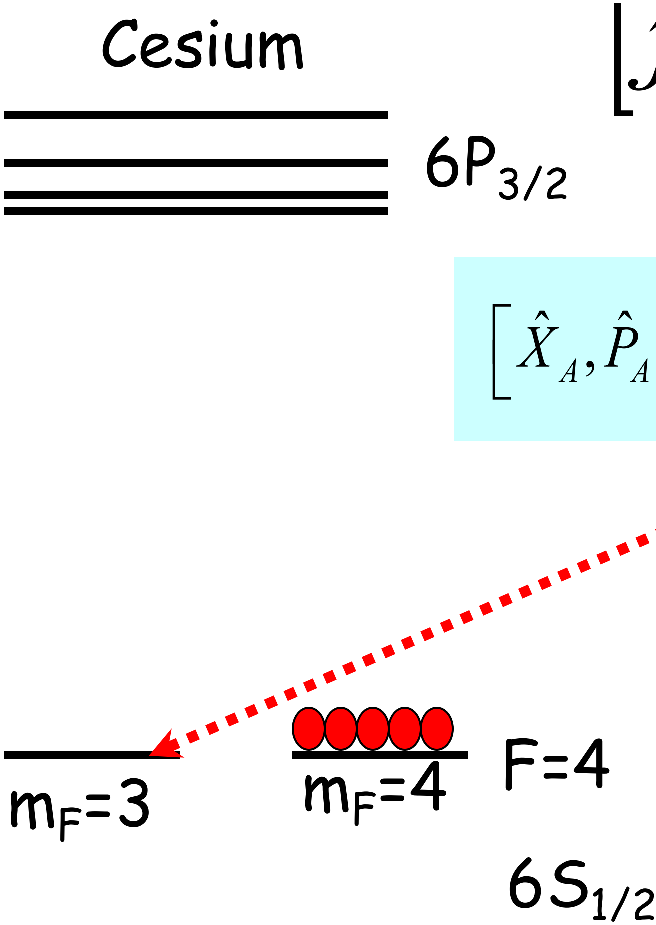
Simon (2000); *Duan, Giedke, Cirac, Zoller* (2000)
Necessary and sufficient condition for entanglement

$$Var(X - X_0) + Var(P + P_0) < 2$$

Canonical collective variables and quantum states for ensemble of 2-level spins (atoms …)

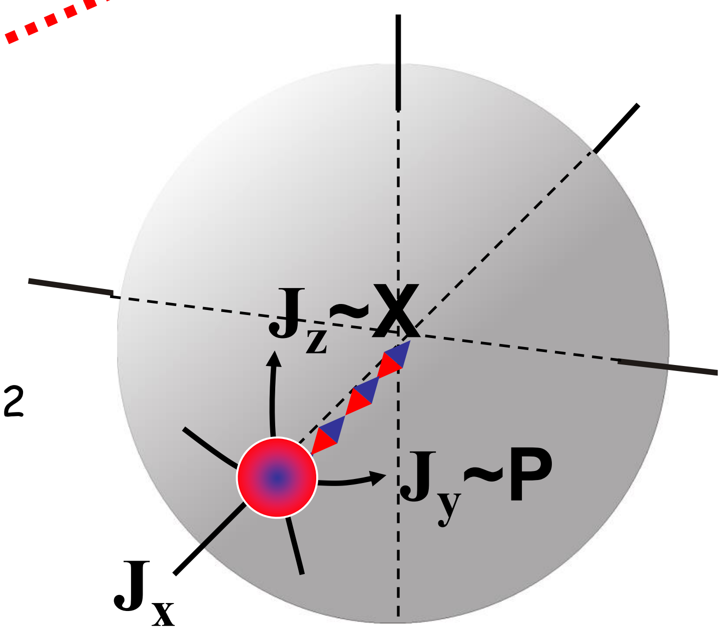
$$\left[\hat{J}_z, \hat{J}_y\right] = iJ_x$$

$$\left[\hat{X}, \hat{P}\right] = i$$

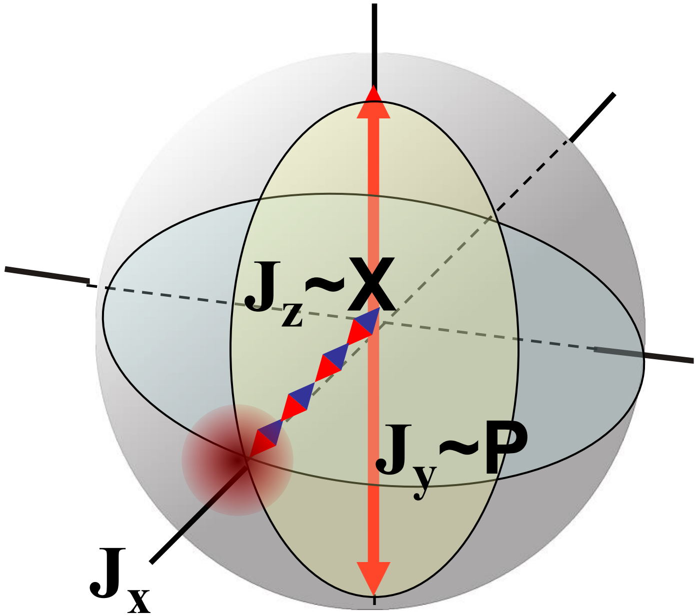


$$\left[\hat{X}_A, \hat{P}_A\right] = i \quad \hat{X}_A = \tfrac{1}{\sqrt{2}}(\hat{b}^\dagger + b) = \frac{\hat{J}_z}{\sqrt{J_x}}, \; P_A = \tfrac{i}{\sqrt{2}}(\hat{b}^\dagger - b) = \frac{\hat{J}_y}{\sqrt{J_x}}$$

$$J = \sum_{i=1}^{N} j_i$$



Ensemble of N polarized atoms = a giant spin



"spin up"

Two levels:
Zeeman splitting,
or hyperfine splitting,
or optical transition
…

"spin down"

$$\left[\hat{J}_y, \hat{J}_z\right] = iJ_x = \tfrac{i}{2} N \qquad \left[\hat{X}, \hat{P}\right] = i$$

$$\hat{X} = \hat{J}_y / \sqrt{J_x}, \qquad \hat{P} = \hat{J}_z / \sqrt{J_x}$$

Uncorrelated atoms:

$$Var\left(J_z\right) = Var\left(J_y\right) = \tfrac{1}{4} N$$

Projection noise

Basic interactions:

Quantum Nondemolition – QND

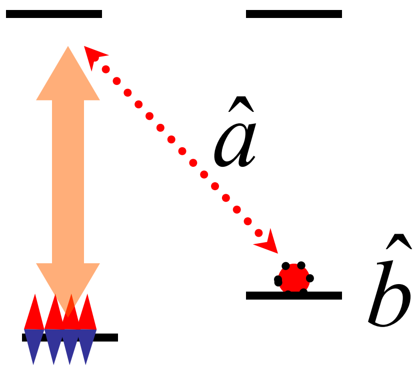
Two-mode entanglement

Beam splitter/Swap operations

K. Hammerer, A. Sørensen, E.P. Reviews of Modern Physics, April 2010
arXiv:0807.3358

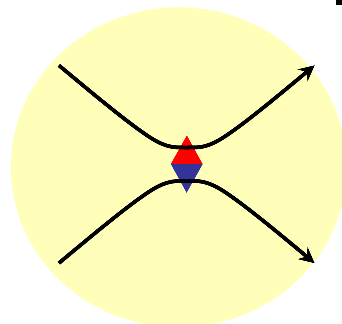
Light-Atoms interface

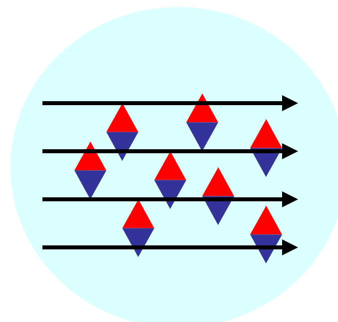
Light-Atoms Entanglement



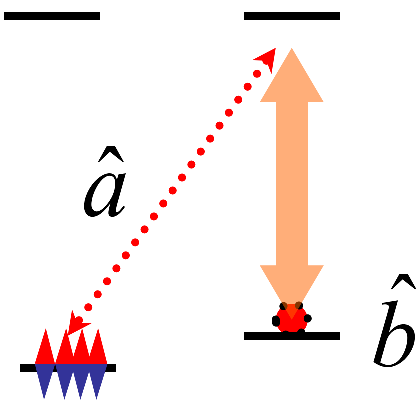
$$H = \chi \hat{a}^{\dagger}\hat{b}^{\dagger} + h.c.$$

Innsbruck, Caltech, Harvard, GIT, Copenhagen, Heidelberg





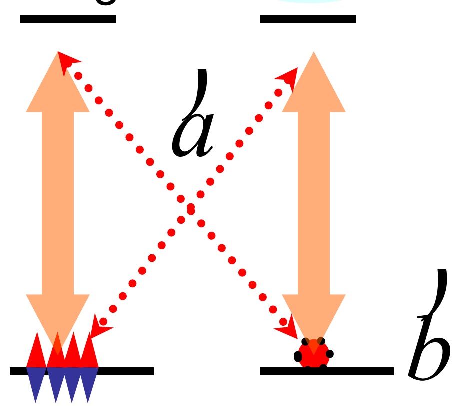
Light-to-Atoms mapping (memory)



$$H = \chi \hat{a}\hat{b}^{\dagger} + h.c.$$

Aarhus, Caltech, Harvard, GIT

Faraday = double Λ interaction= 4wave mixing



Resonant EIT:
Harvard, Caltech, Tokyo, Heidelberg, Georgia Tech

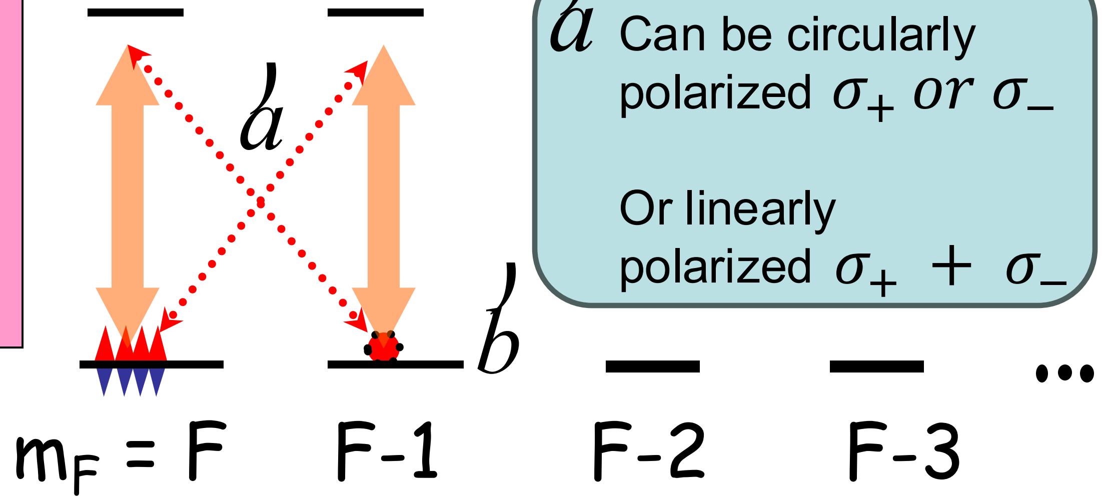
$$H = \chi_{Par}\hat{a}^{\dagger}\hat{b}^{\dagger} + \chi_{BS}\hat{a}\hat{b}^{\dagger} + h.c. \Rightarrow \sqrt{2}\chi \hat{P}_L \hat{P}_A, \; if \; \chi_{Par} = \chi_{BS}$$

Rochester, Copenhagen, Caltech, Kyoto, Arisona…

Atoms
and light
- general off-resonant
interaction

$\hat{a}$ Can be circularly polarized σ_+ *or* σ_-

Or linearly polarized $\sigma_+ + \sigma_-$



$$H = \chi_1 \hat{a}^\dagger \hat{b}^\dagger + \chi_2 \hat{a}\hat{b} + h.c. = k(\hat{P}_L \hat{P}_A + \cancel{\xi^2 \hat{X}_L \hat{X}_A})$$

$$\xi^2 = \frac{\chi_1 - \chi_2}{\chi_1 + \chi_2} = \frac{14 a_2}{a_1}$$

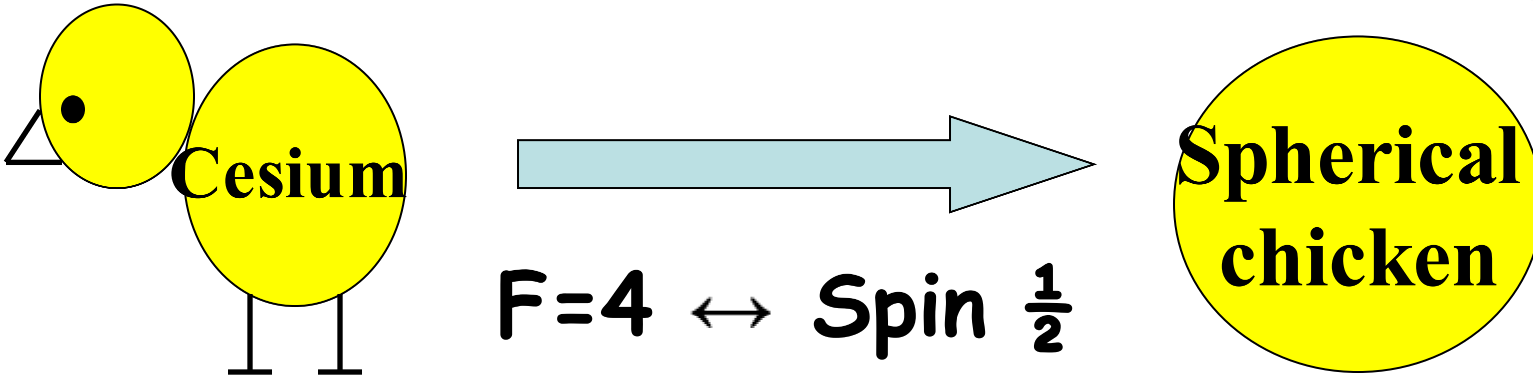
Tensor polarizability (a_2)

Vector polarizability (a_1)

Quantum Nondemolition Interaction limit ↔ tensor term→0:

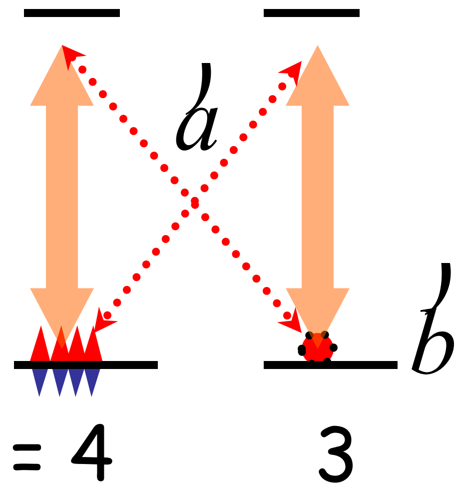
1. For spin ½
2. For alkali atoms, if Δ >> HF of excited state and the interaction time is not too long

Short interaction time → QND



F=4 ↔ Spin ½

QND Interaction



$$H = i\chi\hat{a}^{\dagger}\hat{b}^{\dagger} + i\chi\hat{a}\hat{b}^{\dagger} + h.c. = \sqrt{2}\chi\hat{P}_L\hat{P}_A$$

QND measurement and spin squeezing

$$H \propto \hat{P}_L \hat{P}_A$$

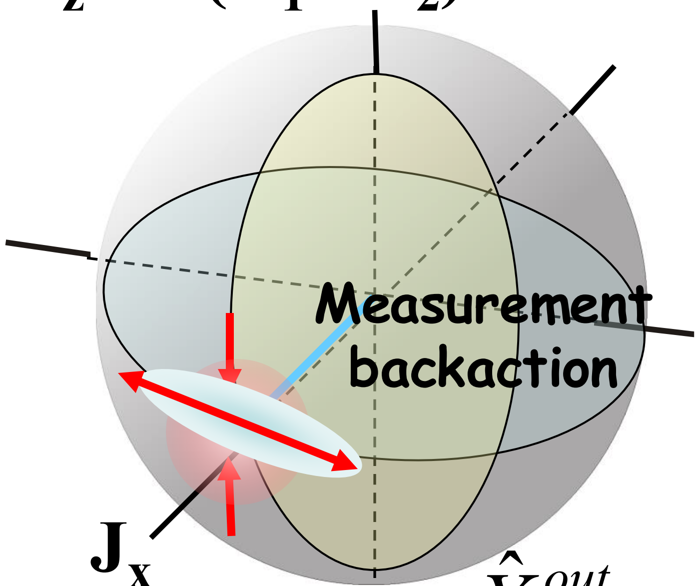
Shot noise of probe light

Probe phase shift Induced by atoms

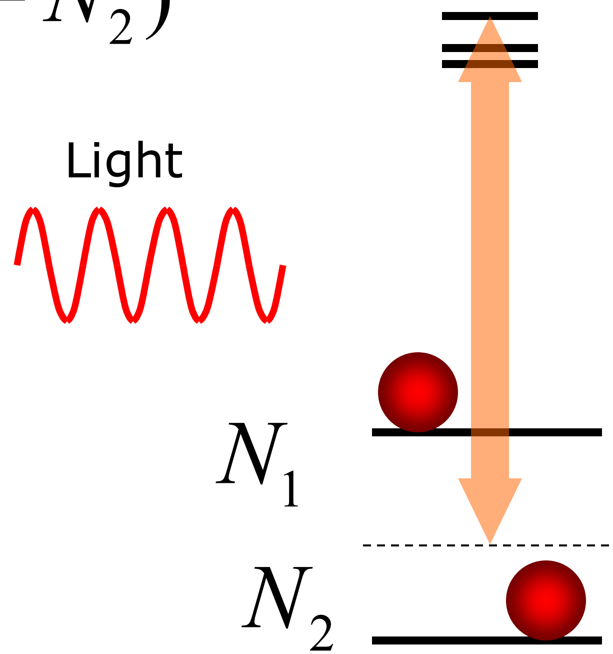
$$\frac{d}{dt}\hat{X}_L = \frac{i}{\hbar}\left[\hat{H}, \hat{X}_L\right] \Rightarrow \hat{X}_L^{out} = \hat{X}_L^{in} + k\hat{P}_A$$

$$\hat{X}_L^{out} = \hat{X}_L^{in} + \frac{k}{\sqrt{N}}(N_1 - N_2)$$

$$\delta J_z^2 = \tfrac{1}{4}(N_1 - N_2)^2 < \tfrac{1}{4}N$$

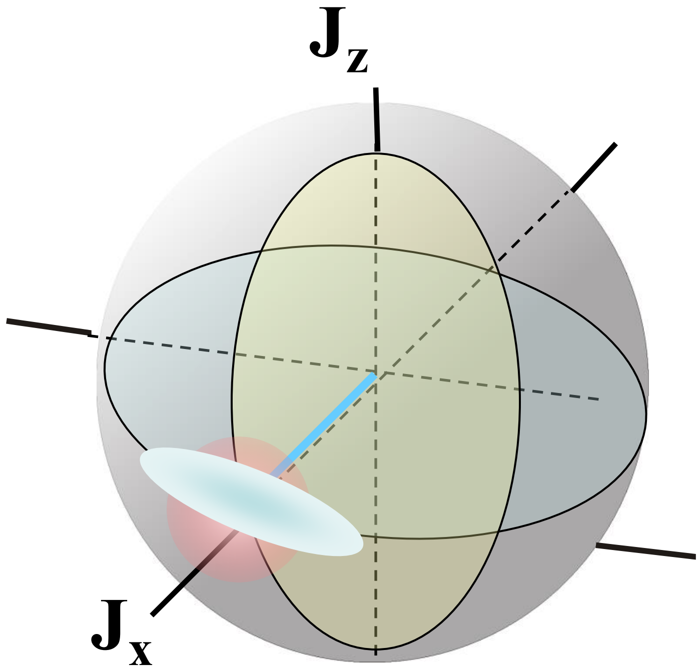


$$\hat{X}_A^{out} = \hat{X}_A^{in} + k\hat{P}_L$$



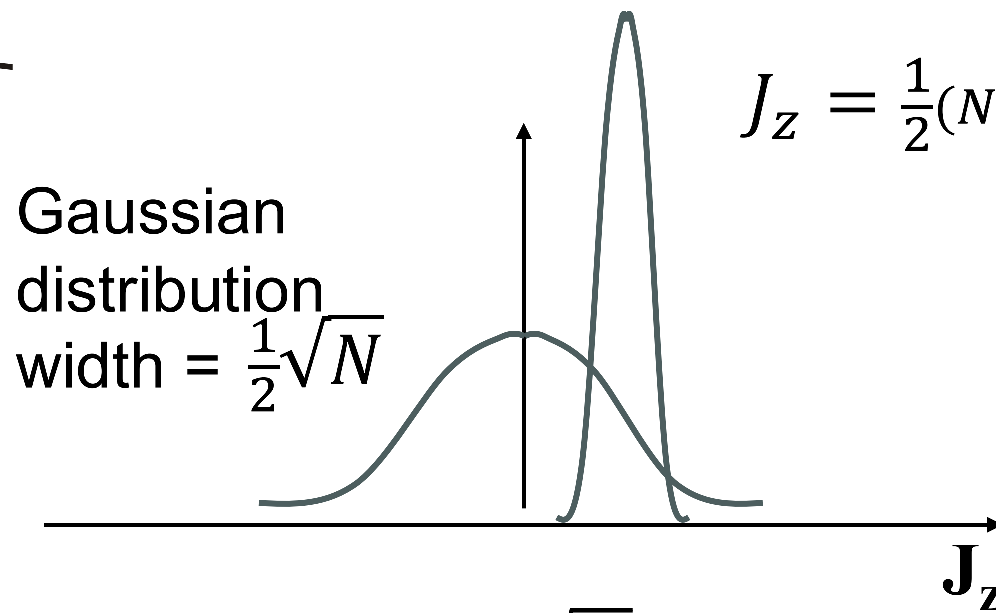
Example: atom clock levels

QND measurement and spin squeezing



$$\hat{X}_L^{out} = \hat{X}_L^{in} + \frac{k}{\sqrt{N}}(N_1 - N_2)$$

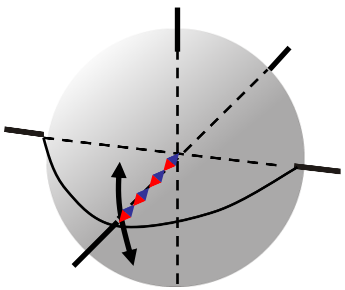
$$J_z = \frac{1}{2}(N_1 - N_2)$$

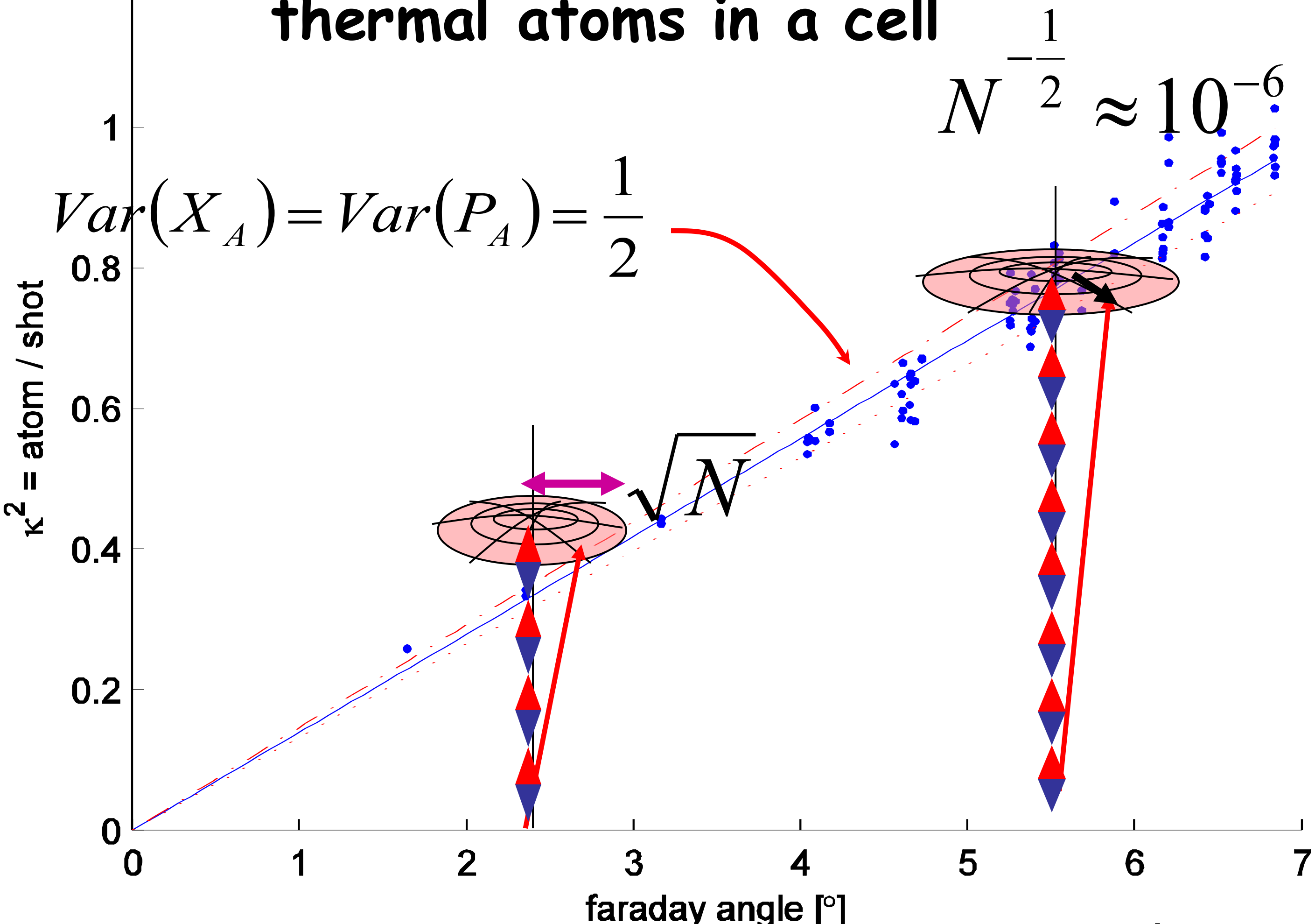


$$\eta = \frac{\kappa^2}{1+\kappa^2} = \frac{\sqrt{d}}{1+\sqrt{d}}$$

Benchmark II: quantum noise of Coherent Spin State $-Var\{J_{z,y}\} \propto N$

Tomography of a coherent spin state (uncorrelated spins) – thermal atoms in a cell





•Shot noise limited detection of light

PLUS

•Stabilization of the atomic noise down to the projection noise

$$\left[\hat{X}_A, \hat{P}_A\right] = i \qquad \hat{X}_A = \tfrac{1}{\sqrt{2}}(\hat{b}^\dagger + b) = \frac{\hat{J}_z}{\sqrt{J_x}}, \; P_A = \tfrac{i}{\sqrt{2}}(\hat{b}^\dagger - b) = \frac{\hat{J}_y}{\sqrt{J_x}}$$

Spin – light coupling rate

$$H_{spin} = \frac{\kappa}{\tau_p} X_{spin} x_{light}$$

$$X_{L\,out} = X_{L\,in} + \kappa P_A$$

Atomic crossection

Optical transition Natural linewidth

Spontaneous Emission probability

$$\kappa^2 \approx \left(\frac{\sigma}{A} N_A\right)\left(\frac{\sigma}{A}\frac{\gamma^2}{\Delta^2} n_{ph}\right) = d_0 \eta \approx 1 \div 3$$

Beam crossection

Atom ' number

Photon ' number

Optical depth

Coherent state -> $\frac{1}{2}\sqrt{N}$

$$\hat{X}^{out}_{light} = \hat{X}^{in}_{light} + \kappa \hat{P}^{in}_{atoms}$$

$$\hat{X}^{out}_{L} = \hat{X}^{in}_{L} + \frac{k}{\sqrt{N}}\underbrace{(N_1 - N_2)}$$

FOM for light-atoms quantum interface – optical depth

$$k^2 \approx \alpha_\Delta s_\Delta \frac{\Delta^2}{\gamma^2}\gamma\tau_{pulse} = \alpha_0 s_\Delta \gamma\tau_{pulse} = \alpha_0\eta$$

Probe depumping parameter spontaneous emission:

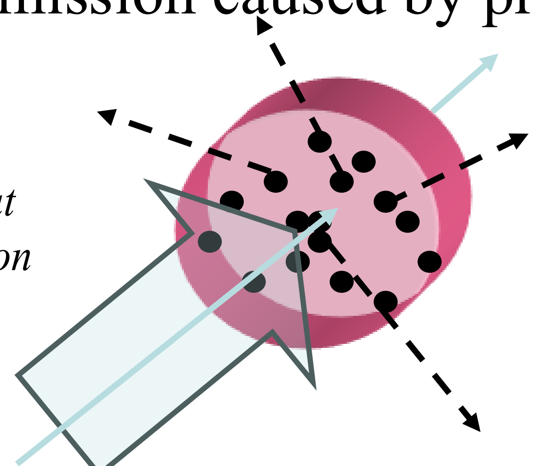
$$\eta << 1$$

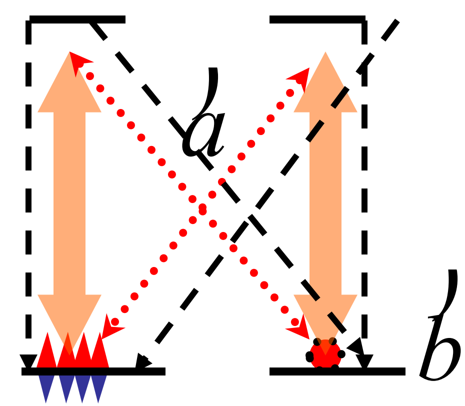
$\alpha_\Delta = \alpha_0\gamma^2/\Delta^2$ optical depth of the atomic sample (absorption coefficient)

$s_\Delta = s_0\gamma^2/\Delta^2$ saturation parameter – the ratio of the Rabi frequency to spontaneous decay rate γ

η probability of spontaneous emission caused by probe pulse

$$k^2 = \alpha_0\eta = \frac{\sigma}{A}N_{at}\eta = \frac{\sigma}{A}m^{scat}_{phon}$$



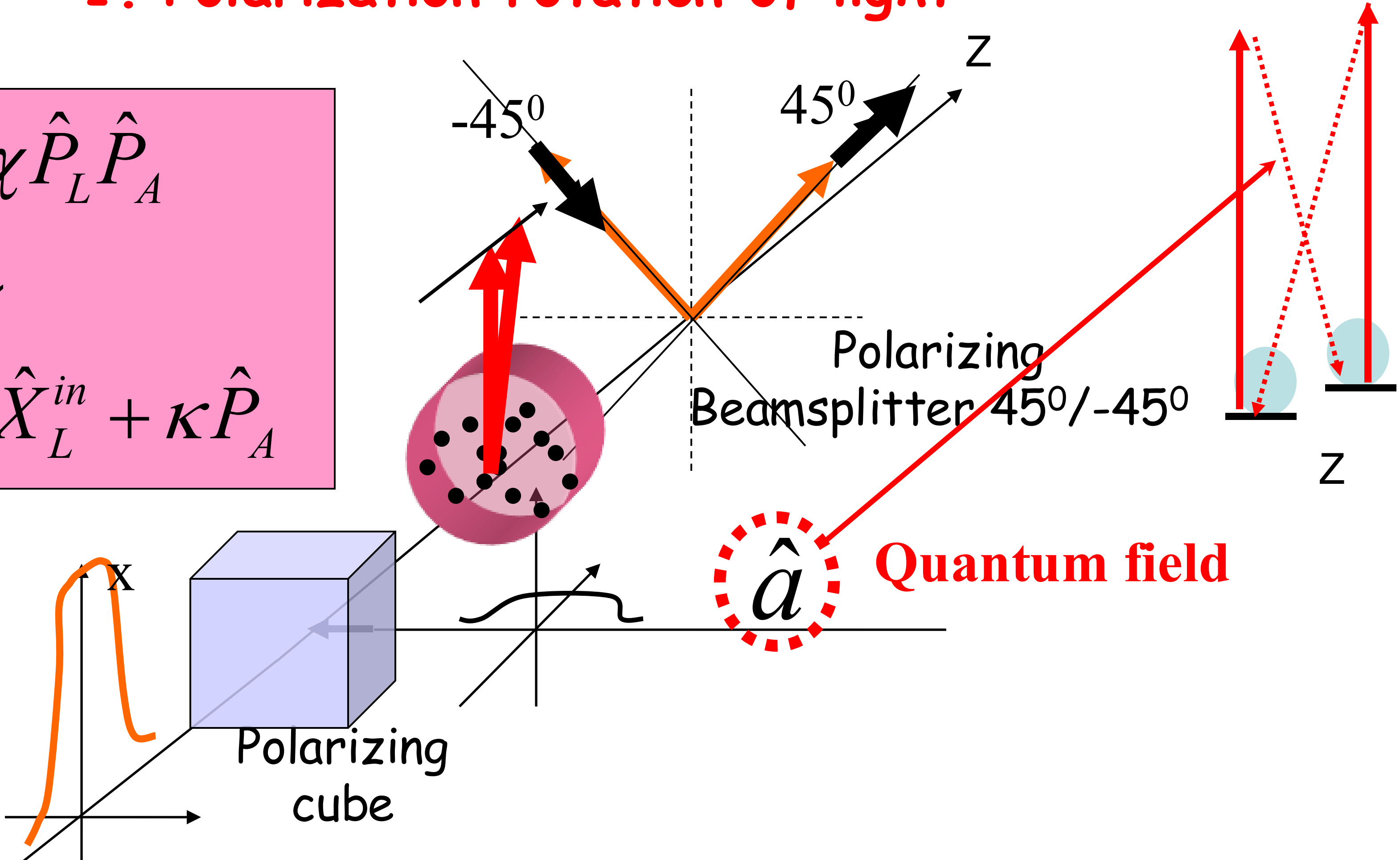


Physics of QND interaction between ensemble and light:
1. Polarization rotation of light

$$\hat{H} = \chi \hat{P}_L \hat{P}_A$$

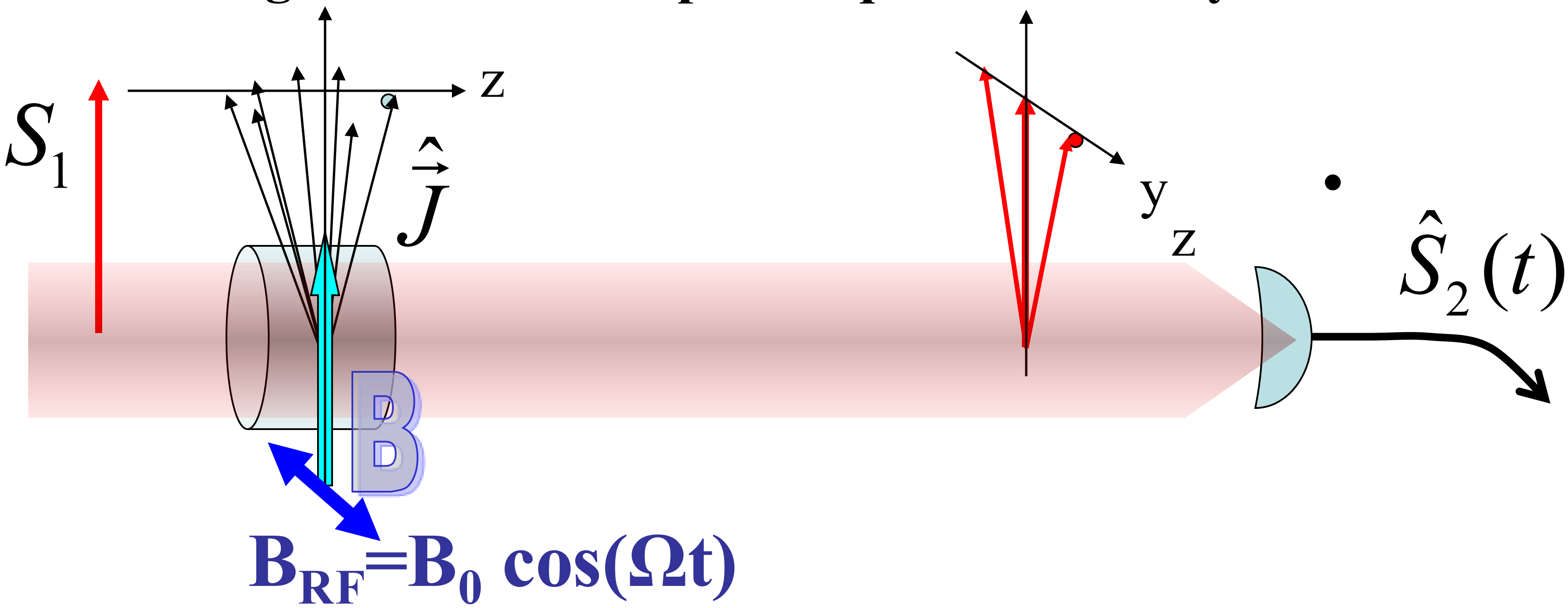
$$\Downarrow$$

$$\hat{X}_L^{out} = \hat{X}_L^{in} + \kappa \hat{P}_A$$



Polarization of light $\quad \hat{S}_2^{out} = \hat{S}_2^{in} + \alpha S_1 \hat{J}_z \rightarrow \hat{\varphi} = \frac{\hat{S}_2^{in}}{S_1} + \frac{\sigma}{A}\frac{\Gamma}{\Delta}\hat{J}_z$

Atomic magnetometer – simplified quantum theory



$$\dot{J}_z = \alpha J_x S_3^{in} \cos \Omega t$$

$$\dot{J}_y = \alpha J_x S_3^{in} \sin \Omega t$$

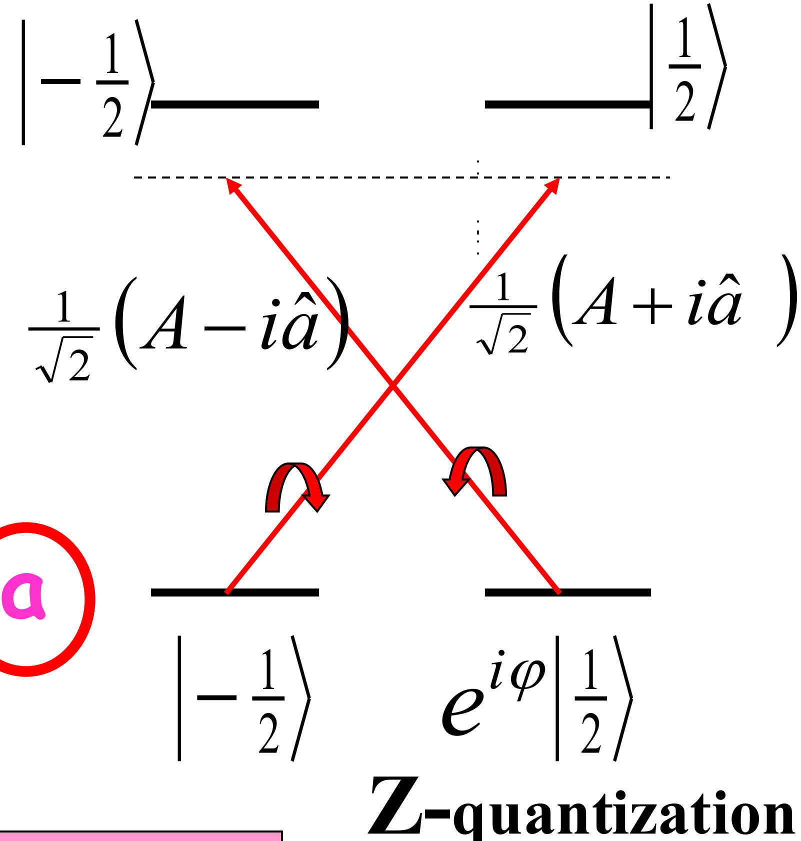
back action of light

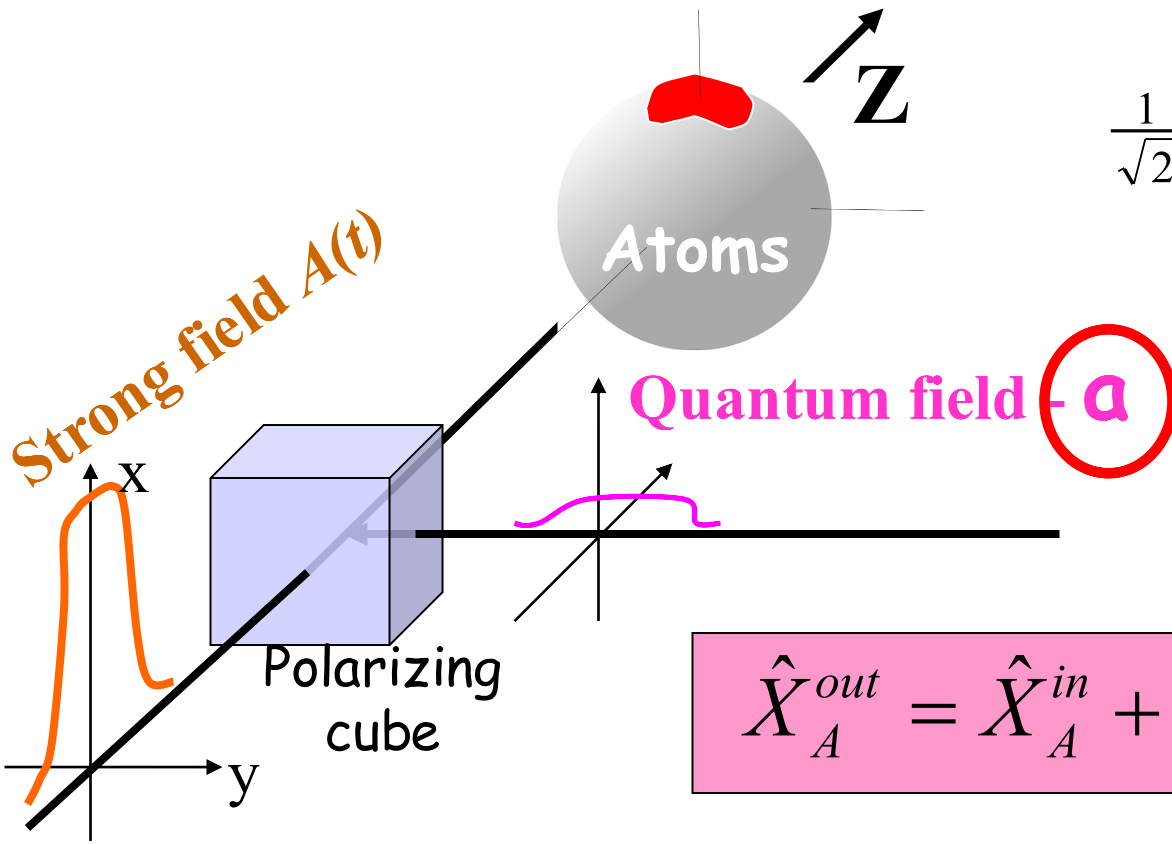
$$\hat{S}_2^{out} = \hat{S}_2^{in} + \alpha \hat{J}_z^{Lab}$$

shot noise

projection noise + signal

Physics of quantum backaction: Dynamic Stark shift of atoms





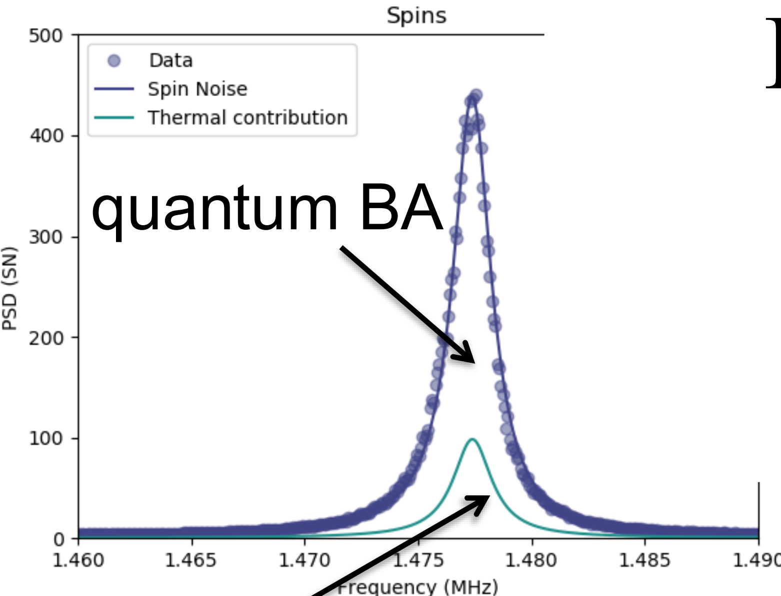
$$\hat{X}_A^{out} = \hat{X}_A^{in} + \kappa \hat{P}_L$$

Atomic spin rotation

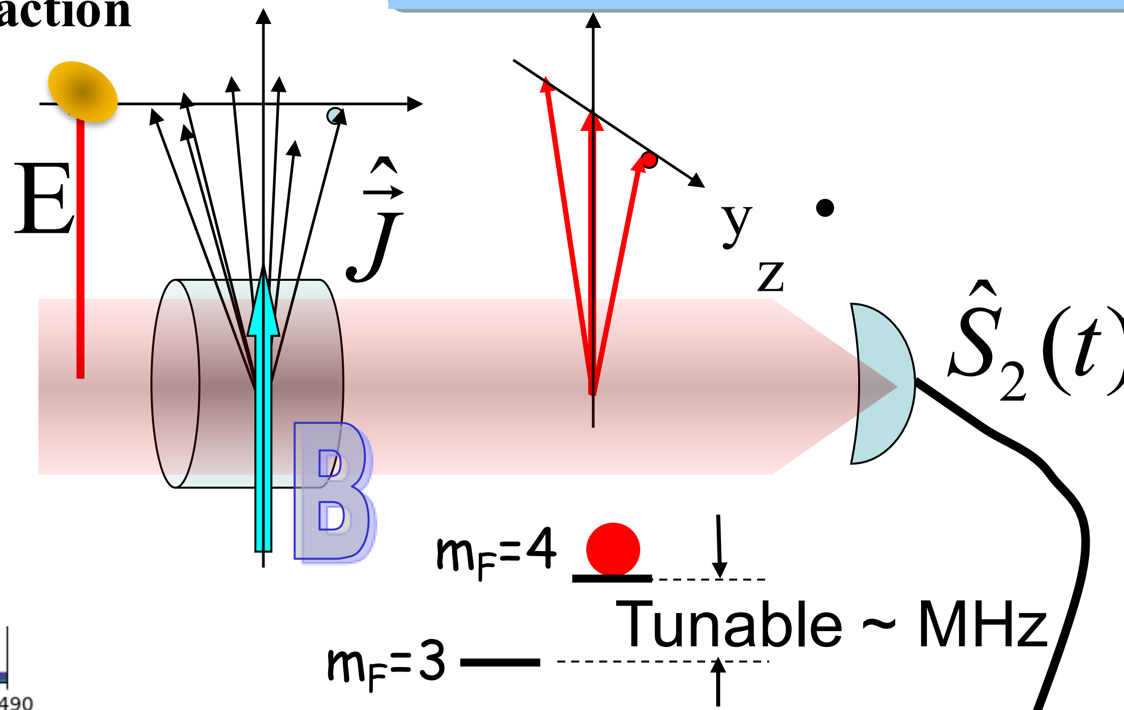
$$\hat{J}_y^{out} = \hat{J}_y^{in} + \alpha J_x \hat{S}_3 \rightarrow \hat{\varphi} = \frac{\hat{J}_y^{in}}{J_x} + \frac{\sigma}{A}\frac{\Gamma}{\Delta}\hat{S}_3$$

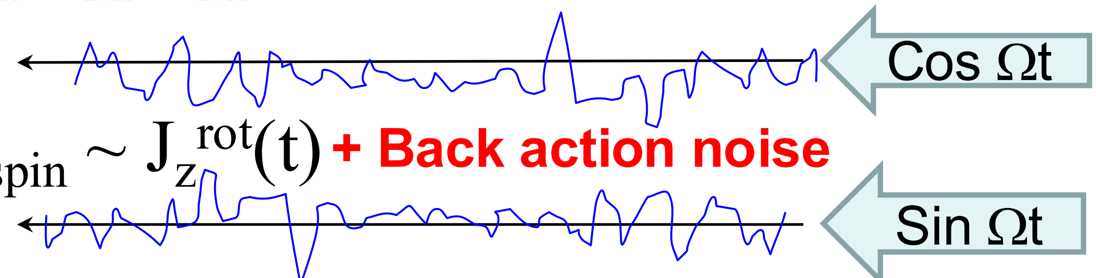
Quantum noise of measurement of spin oscillator: probe noise and backaction

$$J_z^{\,lab} = J_z^{\,rot}\cos\Omega t - J_y^{\,rot}\sin\Omega t$$



(Almost) Ground Spin state





$X_{spin} \sim J_z^{rot}(t)$ **+ Back action noise**

$P_{spin} \sim J_y^{rot}(t)$ **+ Back action noise**

Cooperativity

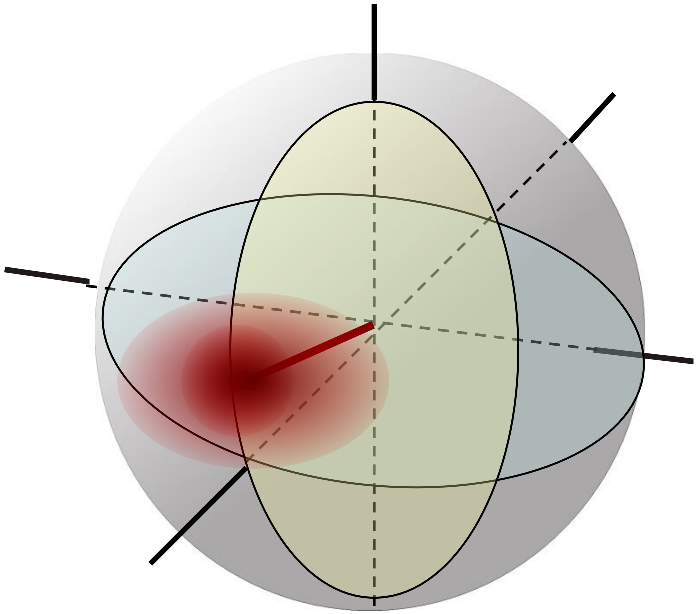
Succeptibility

$$P_{L1,out} = -P_{L1,in} + spin\ state + \Gamma_S \chi_S X_{L1,in}$$

Shot noise of light

Back action of light

Nontrivial problems of quantum measurement



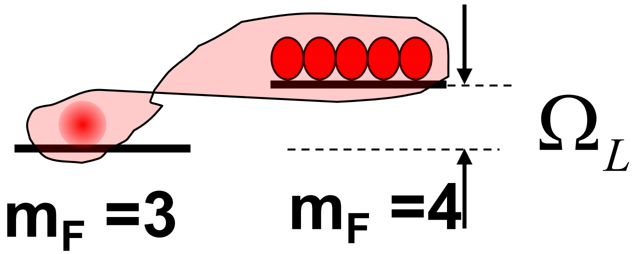
Quantum noise of the initial state of atoms

Quantum measurement changes the state: back action noise of the meter (light)

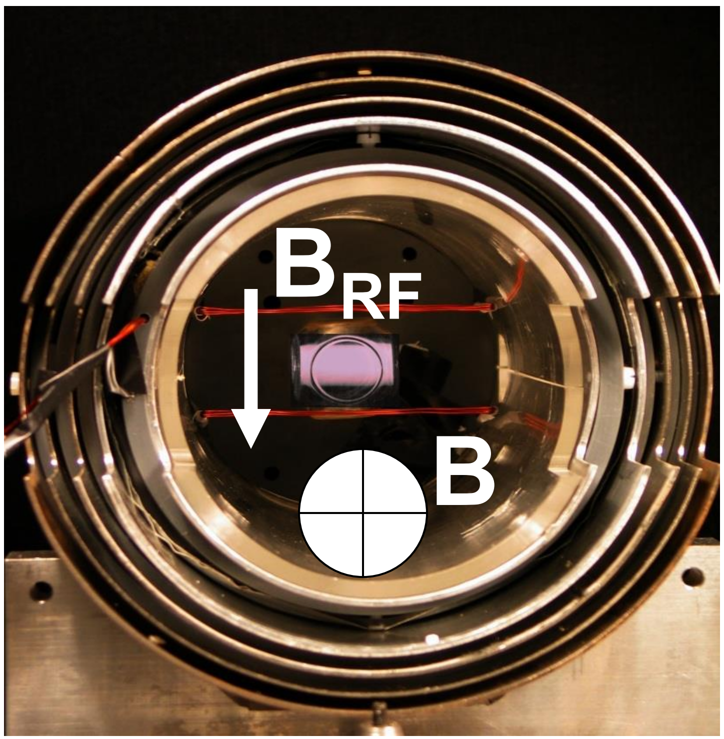
The meter (light) has its own quantum noise which adds to the measurement error

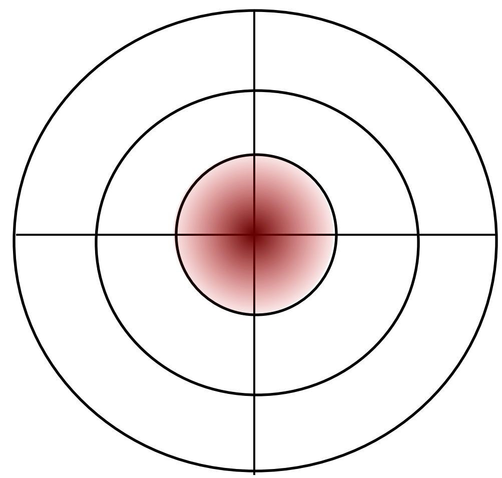
Atomic levels and geometry of experiment

Cesium ground state



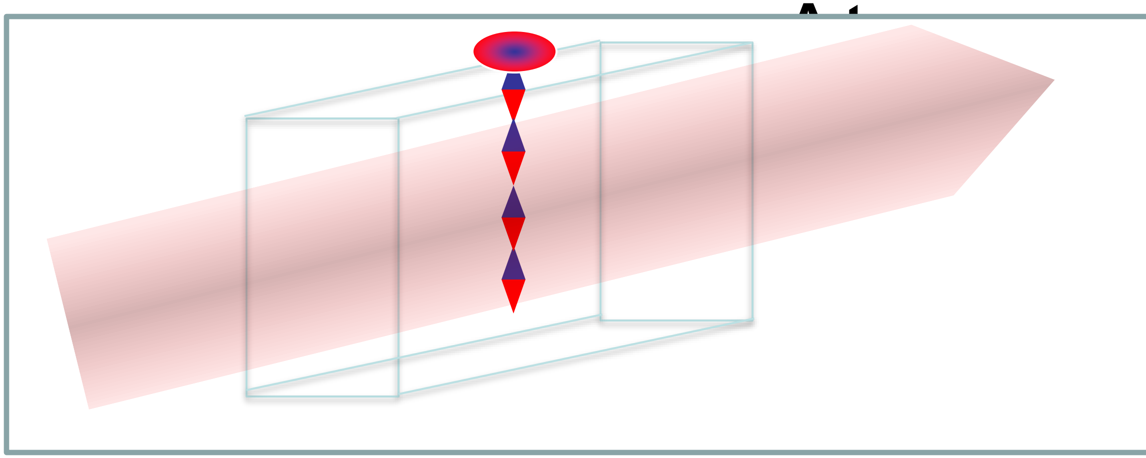
$$B_{RF} = b_{RF} \cos \Omega_L t$$

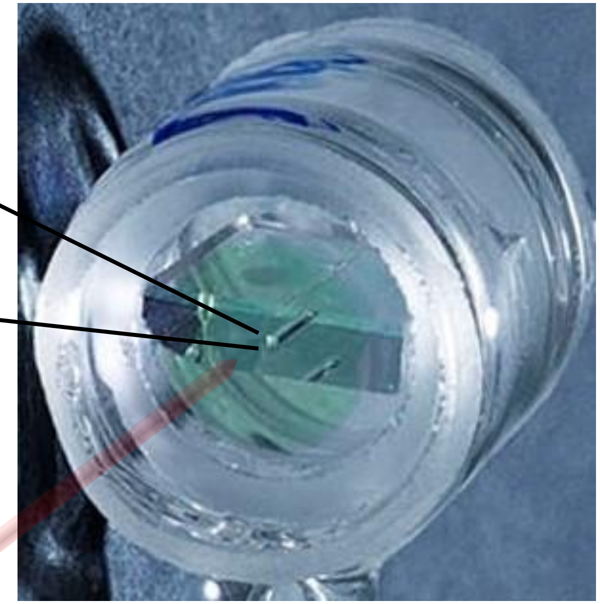




$$\varphi = \gamma b_{RF} T_2$$

Sensor: **collective** spin of 10^9 - 10^{11} Room T



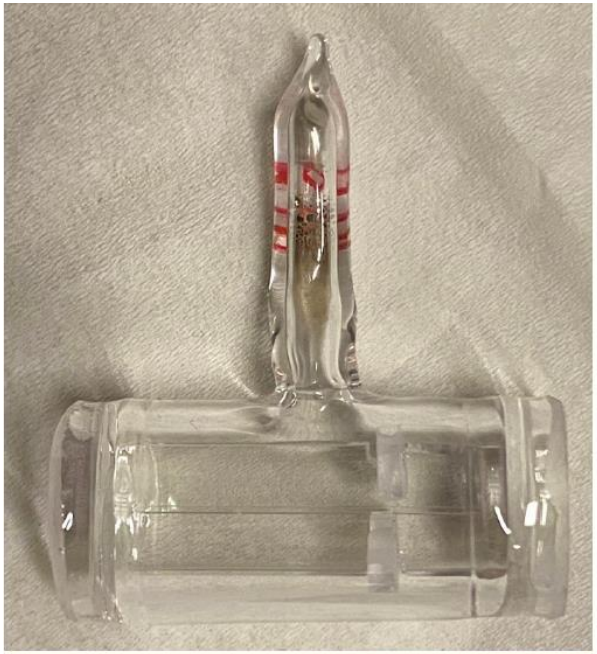


To generate a long-lived collective spin wave:

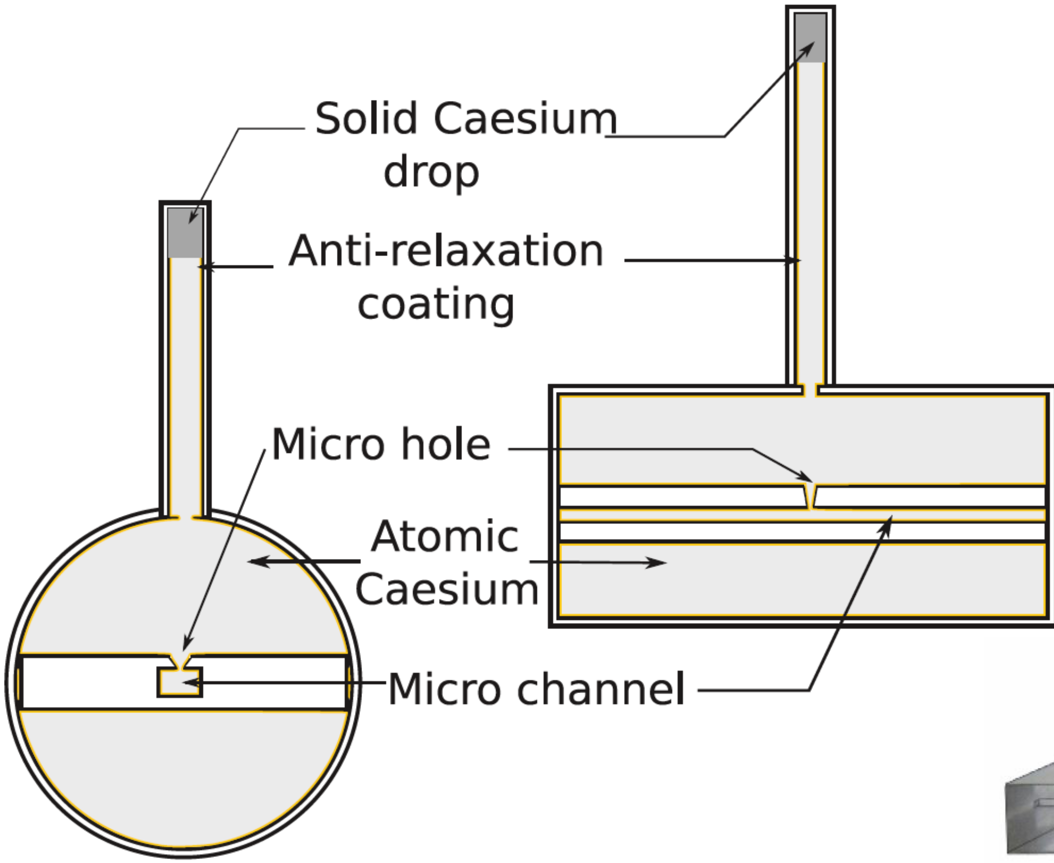
- Light-spin interaction without ”which atom” information
  - Spin protecting coating of cell walls prevents collisional decoherence

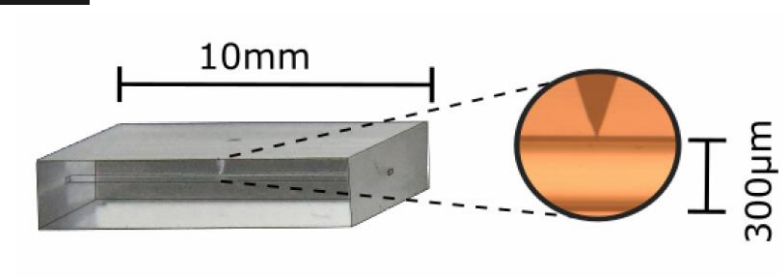
life time of
a spin state ~0.01 – 1 sec at room temperature

Sensor: vapor of room temperature atoms in spin protected environment



Micro cell with anti-relaxation coating





Wall-to-wall flight time $\approx 1\ \mu s$

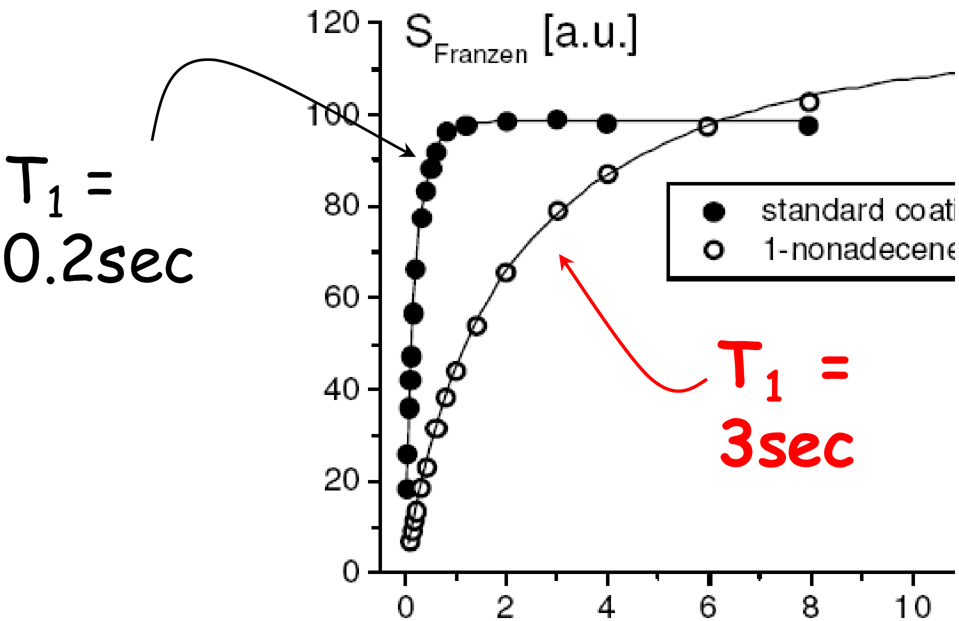
Optics Express 2010

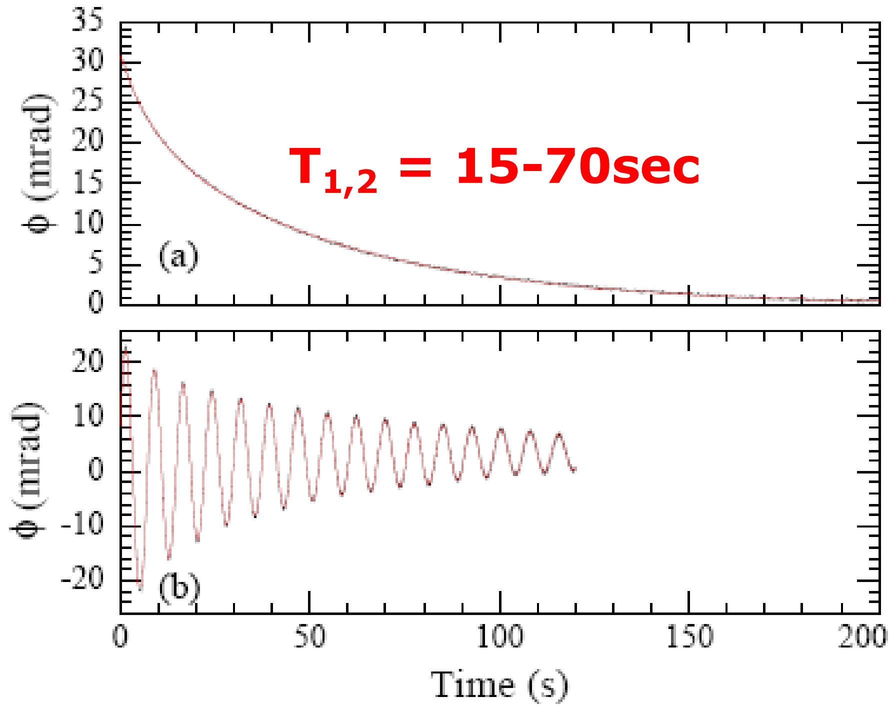
High quality anti-relaxation coating material for alkali atom vapor cells

M. V. Balabas[1,2,*], K. Jensen[1], W. Wasilewski[1], H. Krauter[1], L. S. Madsen[1], J. H. Müller[1], T. Fernholz[1], and E. S. Polzik[1]

1-nonadecene

$$(CH_2 = CH(CH_2)_{16}CH_3)$$





Polarized alkali vapor with minute-long transverse spin-relaxation time

M. V. Balabas,[1] T. Karaulanov,[2] M. P. Ledbetter,[2,*] and D. Budker[2,3]

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(Dated: May 11, 2010)

Summary

Collective excitations and canonical variables for **ensembles** of polarized atoms (spins)

Basic interactions:
QND, two-mode entanglement and swap operations

Quantum limited and **entanglement** assisted **magnetometry**