Quantum Mechanics in Quantum Reference Frame

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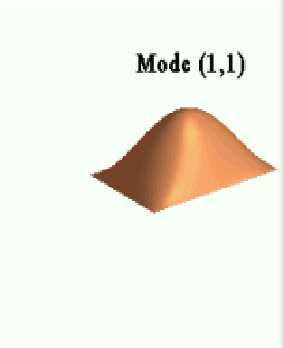
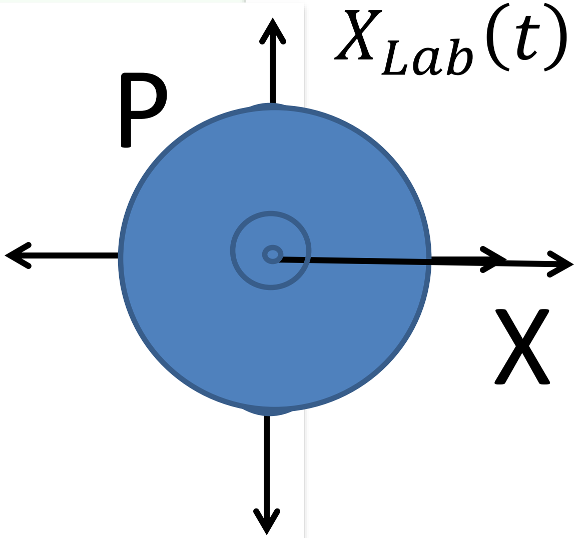
Quantum limits of measurement of motion and fields

$$X_{Lab}(t) = \mathrm{X}\sin(\omega t) + \mathrm{P}\cos(\omega t)$$

$$Var(X)Var(P) \geq 1/4 \qquad [X,P] = i$$



Measuring $X_{Lab}(t)$



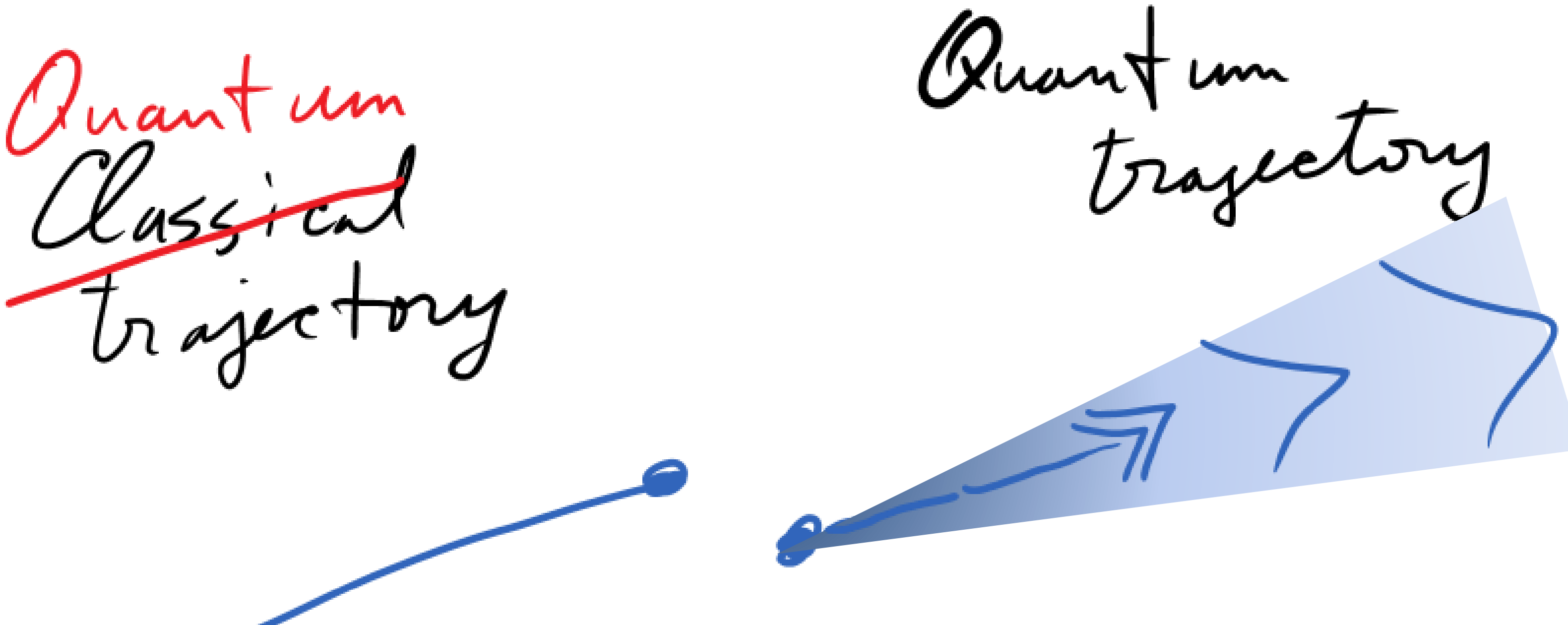
Measurement of X imposes uncertainty on P and the other way around:

Quantum Back Action of measurement

And yet, arbitrary small perturbations in BOTH position and momentum can be measured simultaneously

Trajectories without quantum uncertainties
in a negative mass reference frame

AKA quantum-mechanics-free subspaces



“Entanglement between distant macroscopic mechanical and spin systems”.
R. Thomas *et al.* **Nature Phys.** 17, 228–233(**2021**).
“Quantum back-action-evading measurement of motion in a negative mass reference frame”
C. Moller et al, **Nature** 22980 **(2017)**
“Establishing Einstein-Podolsky-Rosen channels between nanomechanics and atomic ensembles”. K. Hammerer, M. Aspelmeyer, ESP, P. Zoller. **PRL** 102, 020501 (**2009**).
“Trajectories without quantum uncertainties”. K. Hammerer and ESP, **Annalen der Physik . (2015)**

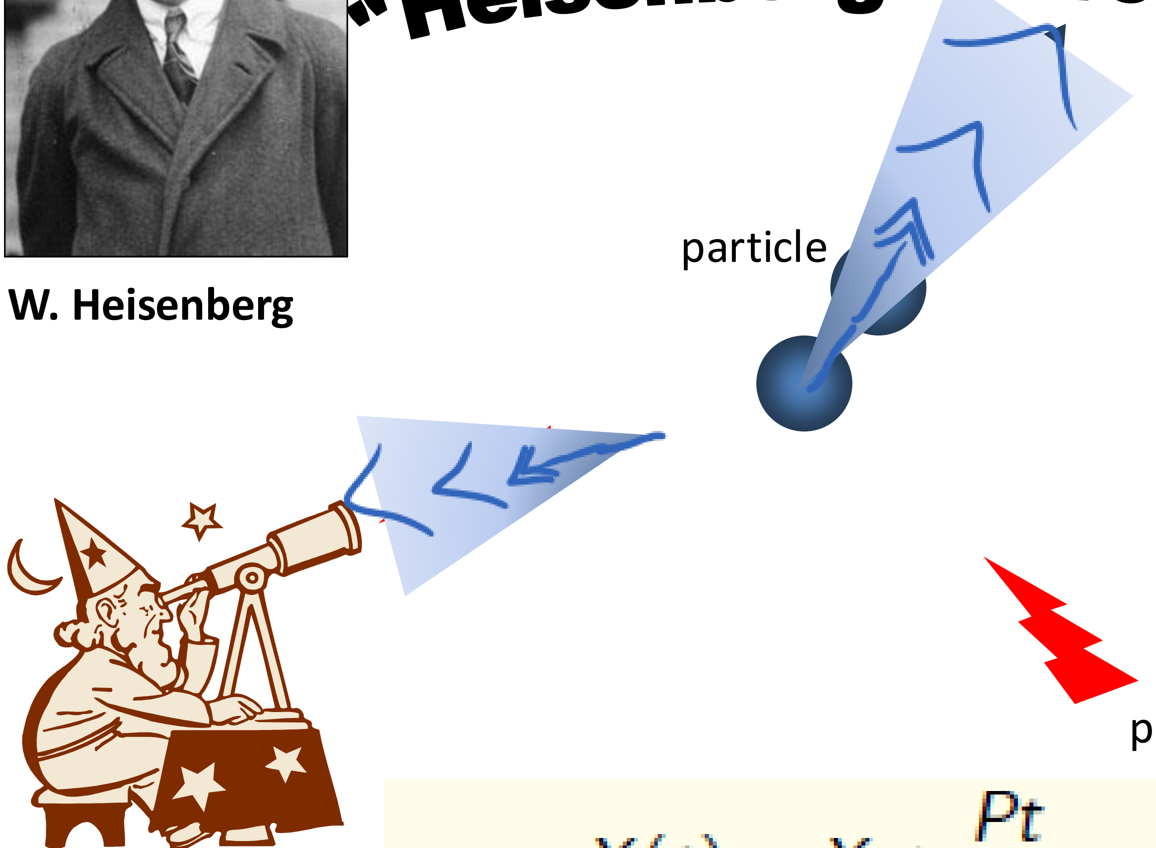
See also Tsang and Caves. Quantum mechanics free subspaces, PRL 2010.

Standard quantum limit example: displacement measurement

"Heisenberg microscope"

W. Heisenberg

N. Bohr



$$X(t) = X + \frac{Pt}{m}, \quad \Delta X \Delta P \geqslant \frac{\hbar}{2} \Rightarrow$$

$$[\Delta X(t)]^2 \geqslant (\Delta X)^2 + \frac{\hbar^2 t^2}{4m^2(\Delta X)^2} \geqslant \frac{\hbar t}{m} \quad \text{(SQL)}$$

Measurement of motion beyond SQL in a negative mass reference frame

1. Define trajectory relative to a quantum reference

2. Reference system has an effective negative mass

3. Entangled state of the reference and the probed systems is generated

4. Measurement (relative to reference frame)

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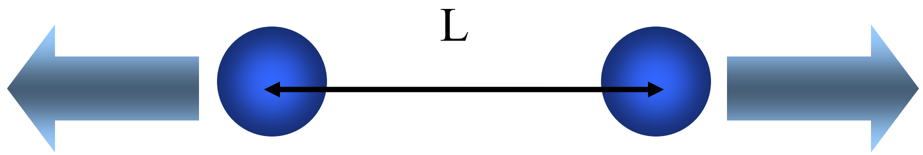
Einstein-Podolsky-Rosen (EPR) entanglement 1935

2 particles entangled
in position/momentum

$$\left[\hat{X}_1-\hat{X}_2,\hat{P}_1+\hat{P}_2\right]=0$$



$\hat{X}_1,\hat{P}_1$ $\hat{X}_2,\hat{P}_2$

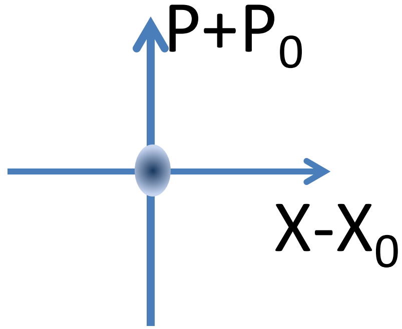
$$\hat{X}_1-\hat{X}_2=L \qquad \hat{P}_1+\hat{P}_2=0$$

Simon (2000); *Duan, Giedke, Cirac, Zoller* (2000)

<u>Necessary and sufficient condition for entanglement</u>

$$Var(X-X_0)+Var(P+P_0)<2$$

Trajectory in a quantum reference frame with negative mass



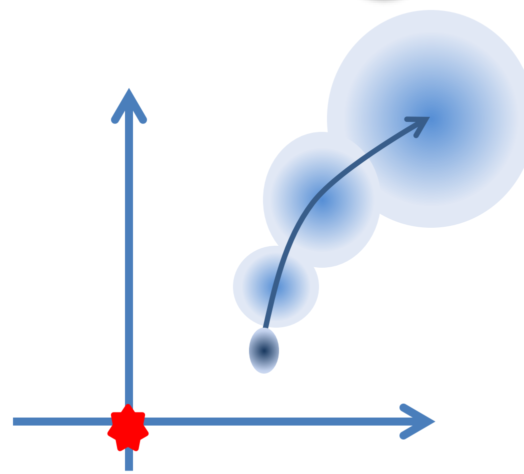
$$X - X_0 = X_{X0} \to 0$$
$$P + P_0 \to 0$$

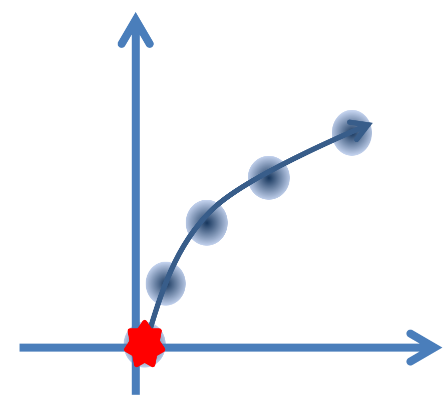
Probe system entangled with origin system

$$X(dt)_{X0} = X(0)_{X0} + \left(\dot{X} - \dot{X}_0\right)dt$$
$$= X(0)_{X0} + (P + P_0)dt$$

$m = - m_0 = 1$

Not good enough





$$X(t) = X(0)\cos(\omega t) + P(0)\sin(\omega t)$$

Joint measurement of motion of two oscillators, one with negative mass and frequency (ω = - ω_R):

Reference oscillator with negative mass, frequency, and k

$$\omega^2 = {}^{k}/_{m}$$

$$X(t) - X_R(t) =$$
$$[X(0) - X_R(0)]\cos(\omega t) + [P(0) + P_R(0)]\sin(\omega t)$$

EPR:

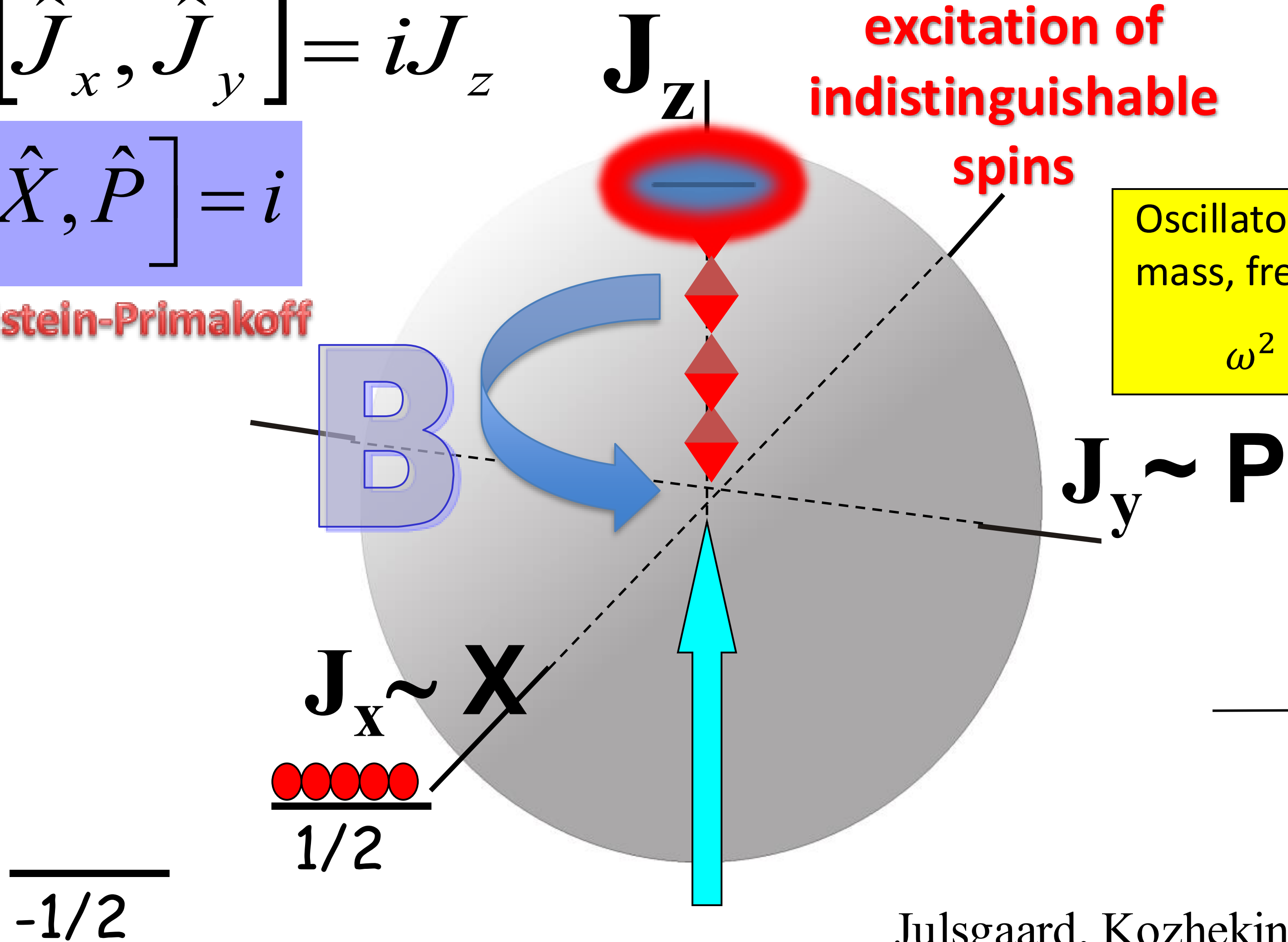
$$Var(X - X_0) + Var(P + P_0) < 2$$

Spin ensemble = positive/negative mass oscillator

$$\left[\hat{J}_x, \hat{J}_y\right] = iJ_z$$

$$\left[\hat{X}, \hat{P}\right] = i$$

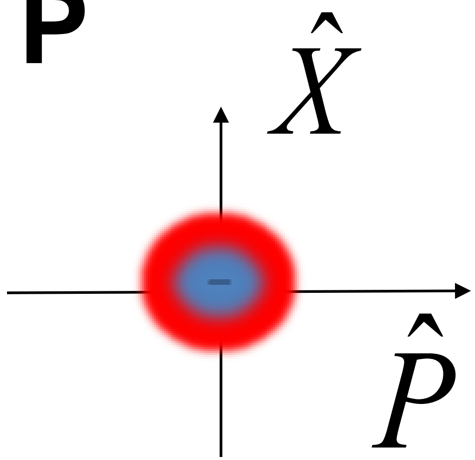
Holstein-Primakoff



$$J = \sum_{i=1}^{N} j_i$$

Oscillator with negative mass, frequency, and k

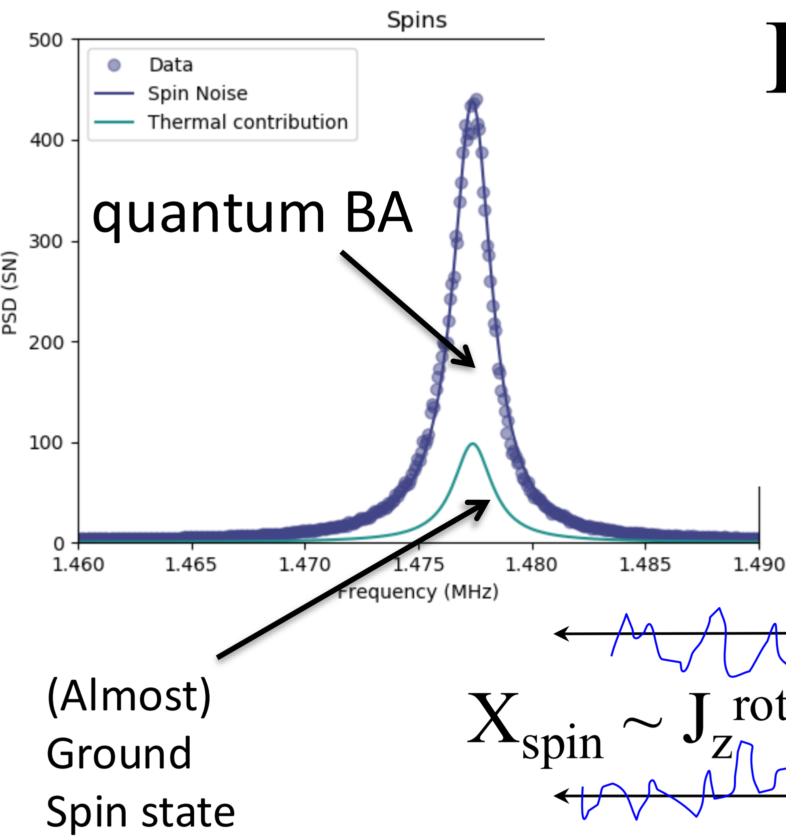
$$\omega^2 = k/m$$

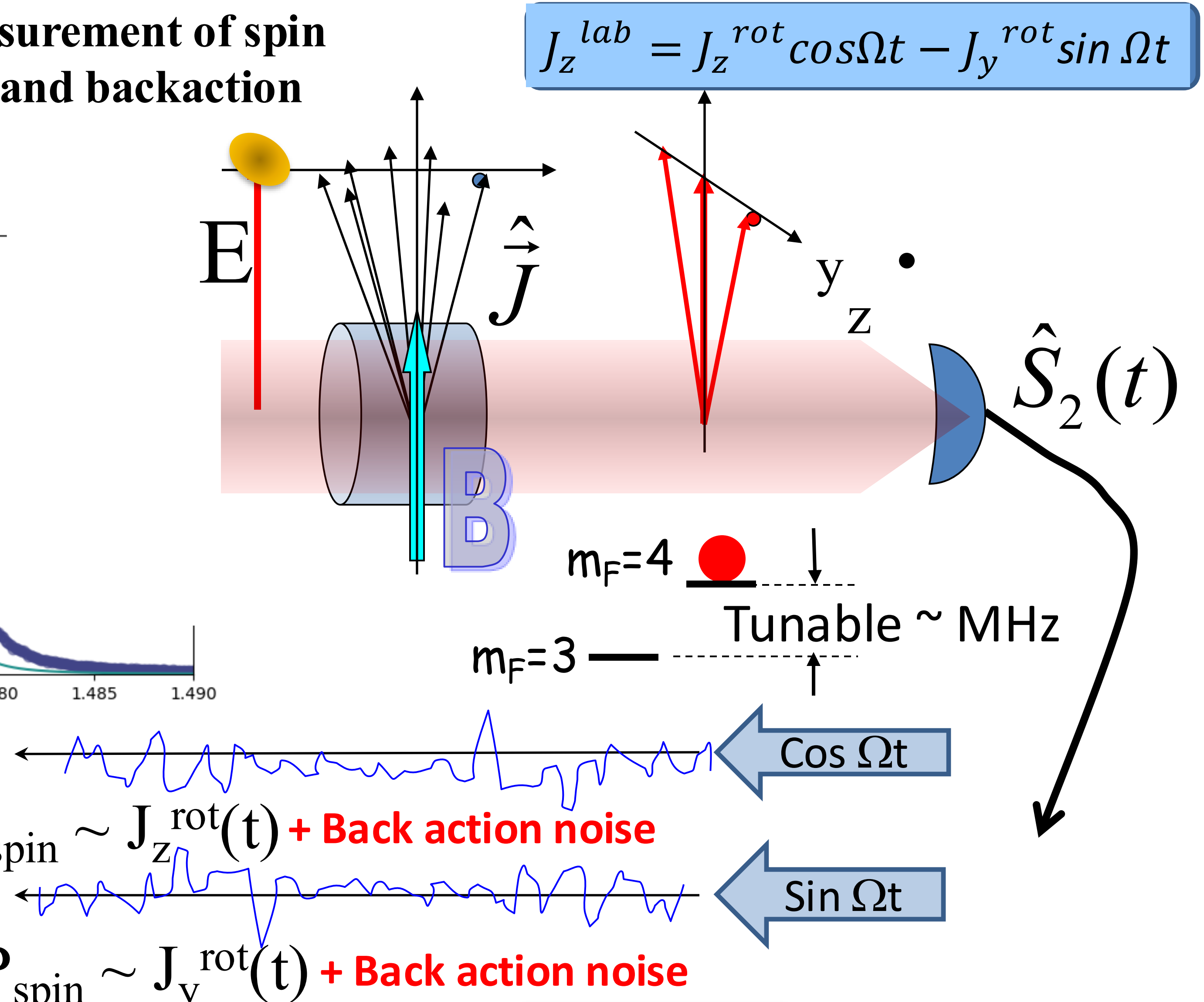


Julsgaard, Kozhekin, ESP, Nature **2001**
Dideriksen et al. Nature Comm. **2021**

$$\left|-\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\ldots\right\rangle + \left|\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2}\ldots\right\rangle + \left|\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}\ldots\right\rangle + \ldots$$

Quantum noise of measurement of spin oscillator: probe noise and backaction





$$P_{L1,out} = -P_{L1,in} + spin\ state + \Gamma_S \chi_S X_{L1,in}$$

Shot noise of light

Back action of light

Application of measurement in negative mass reference frame #1

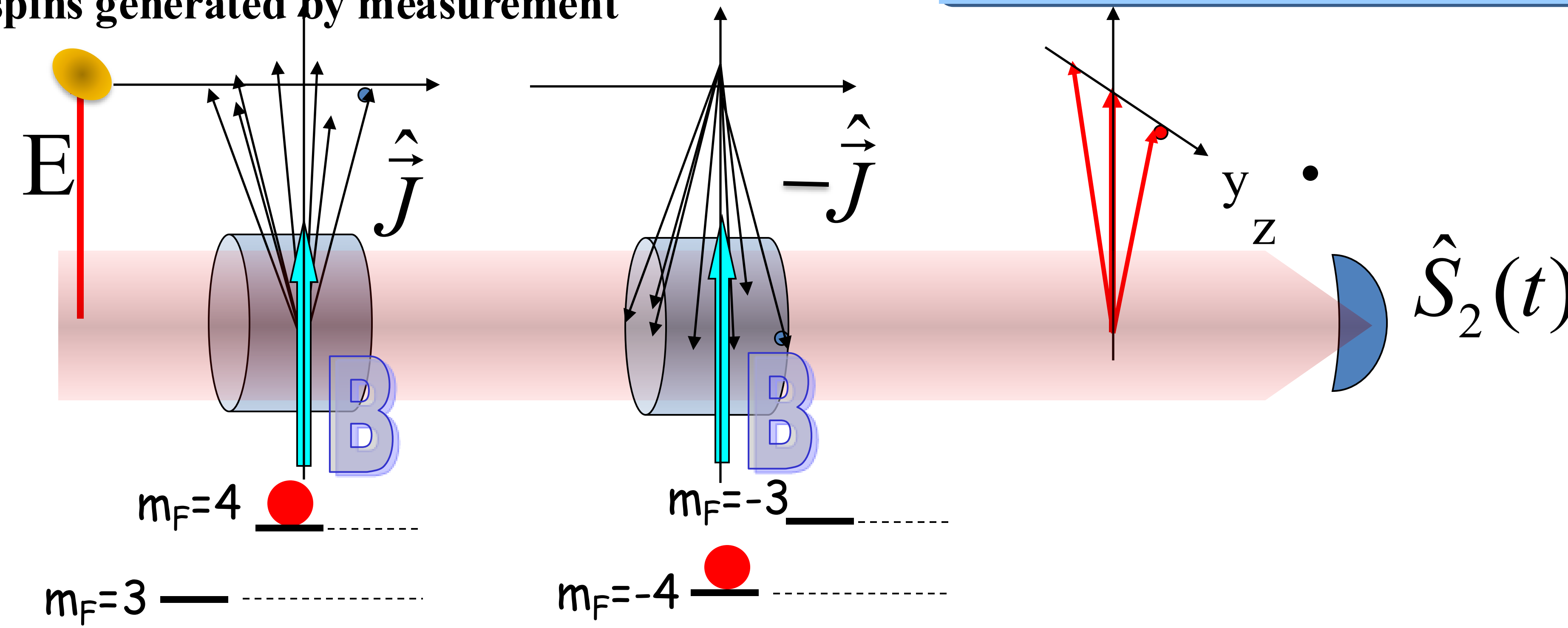
Quantum limited and **entanglement** assisted **magnetometry** with 10^{-16} Tesla/√Hz sensitivity

- Two magnetic sensors with opposite spin orientations
- Generation of entanglement of two sensors
- Magnetic field applied and measured

Phys.Rev.Lett. 104, 133601 (2010) [arXiv:0907.2453]
W. Wasilewski, K. Jensen, H. Krauter, J.J. Renema, M. V. Balabas, E.P.

Quantum entanglement of two macroscopic spins generated by measurement

$$J_z^{lab} = J_z^{rot}\cos\Omega t - J_y^{rot}\sin\Omega t$$



Equal cooperativities

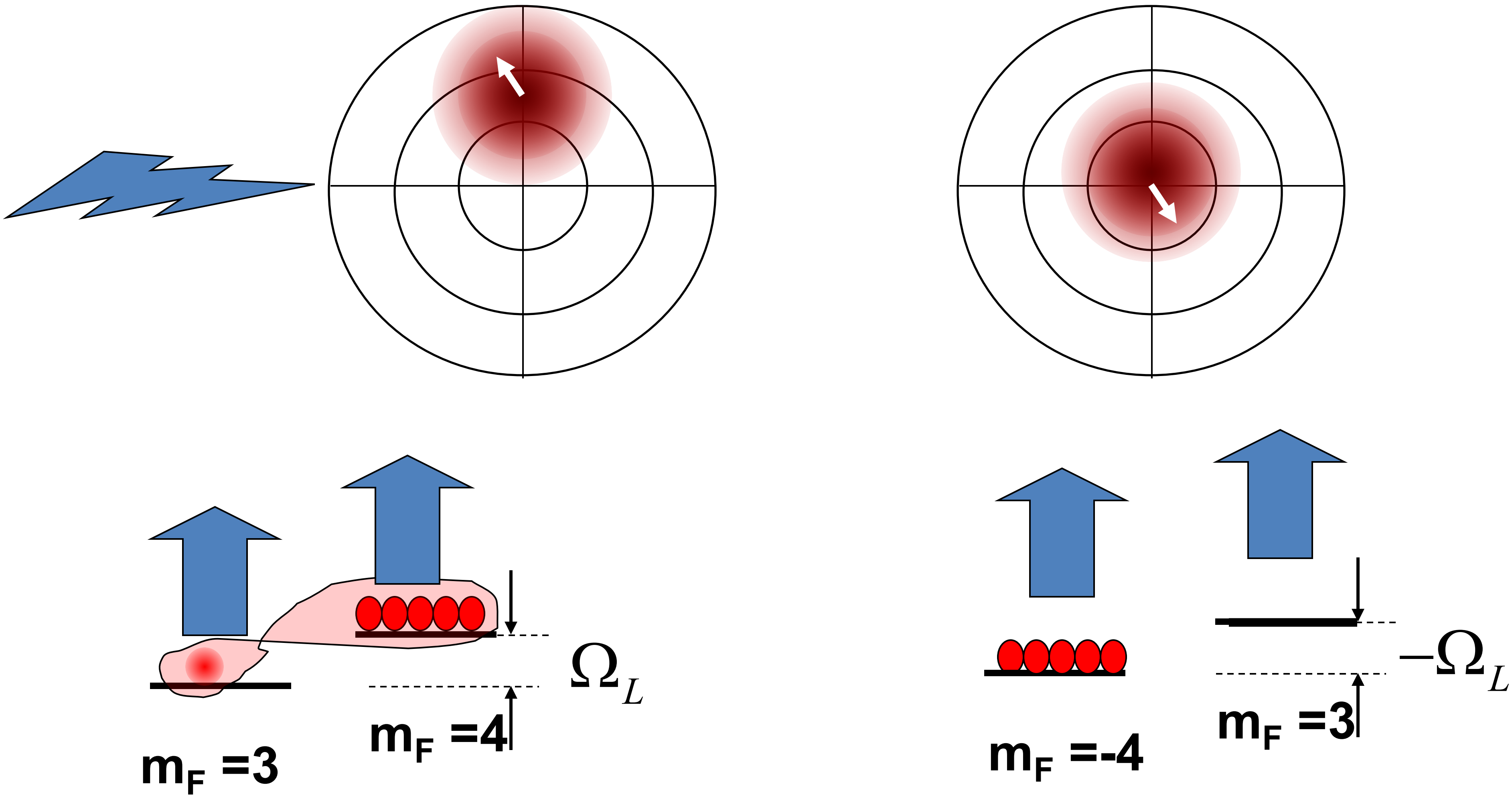
$$P_{L,out} = -P_{L,in} + \sqrt{\Gamma_S\gamma_S}\chi_S(J_{z1}^{rot} + J_{z2}^{rot}) + \Gamma_S\chi_S X_{L1,in} - \Gamma_S\chi_S X_{L1,in}$$

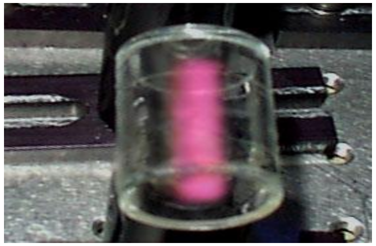
Shot noise of light

Opposite sign succeptibilities

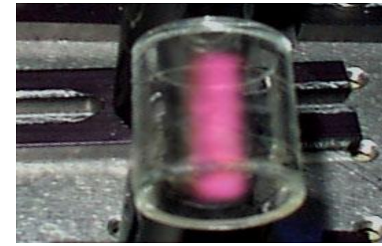
Back action of light cancelled out in the measurement

Quantum back action of probe light on atoms: calcellation via entanglement of two ensembles





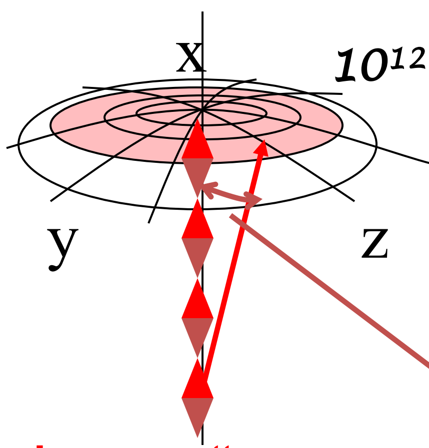
Entanglement of two macroscopic objects.



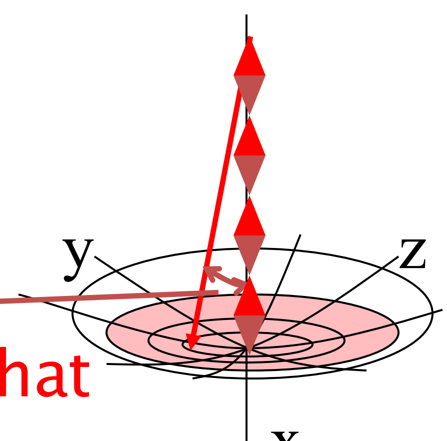
$$Var\left(\hat{J}_{z1}+\hat{J}_{z2}\right)/2J_x+Var\left(\hat{J}_{y1}+\hat{J}_{y2}\right)/2J_x<1$$

Can be created *by a measurement*

10^{12} spins in each ensemble



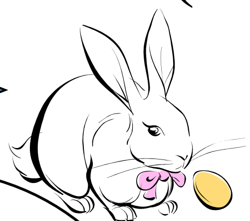
$\sim N^{-\frac{1}{2}}$

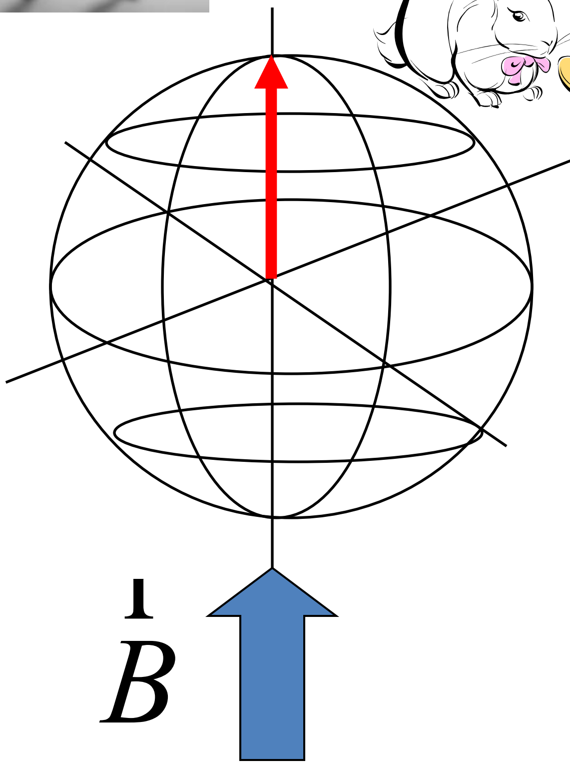


Spins which are “more parallel” than that are entangled

Detection of tiny oscillating magnetic fields

$$\vec{B}_{RF} = b\cos(\Omega t)$$



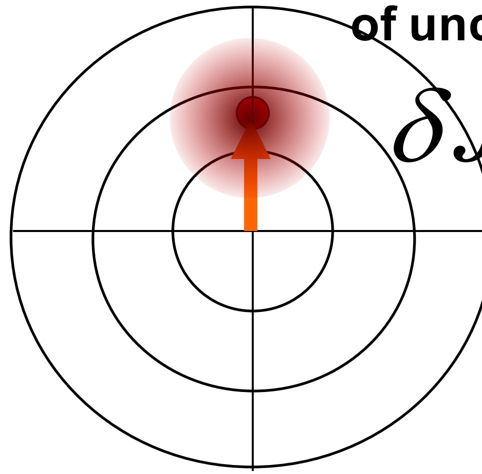


$\vec{B}$

Bias magnetic field

Larmor frequency Ω

Spin dynamics top view



Minimal quantum noise of uncorrelated spins

$$\delta J_{\perp} = \sqrt{N_A}$$

$$J_{\perp} \approx \gamma B_{RF} N_A T_2$$

γ – Gyromagnetic constant

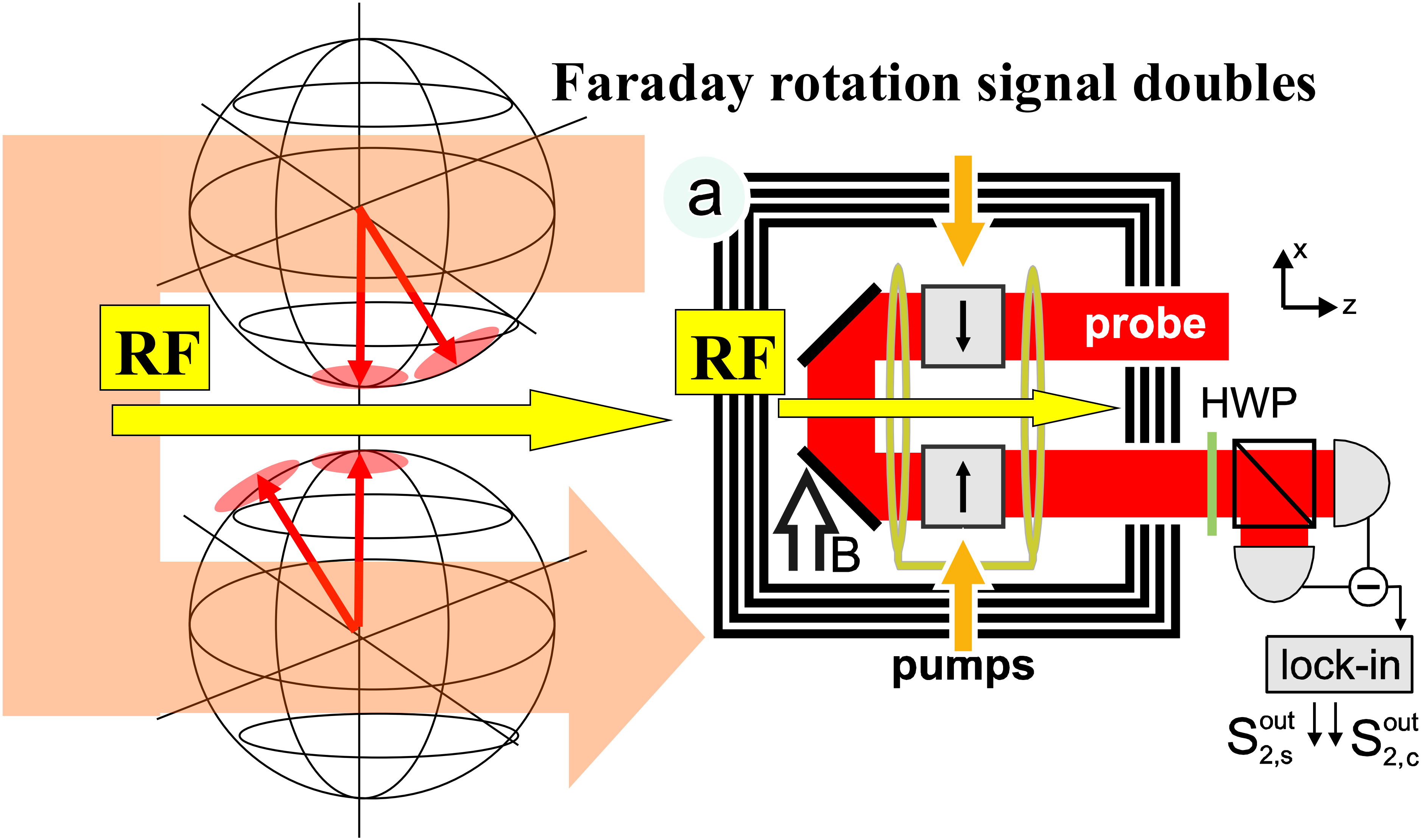
T_2 – Transverse spin coherence time

Calcellation of measurement back action with two cells

$$\left[\hat{J}_{z1}+\hat{J}_{z2},\hat{J}_{y1}+\hat{J}_{y2}\right]=i(\hat{J}_{x1}+\hat{J}_{x2})=0$$

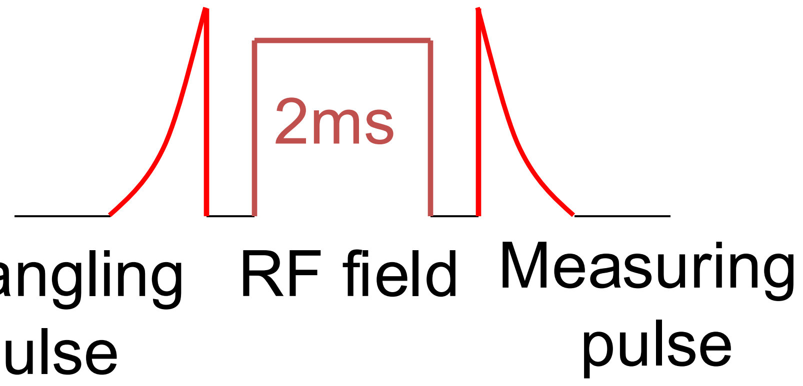
measurement does not change relative spin orientation

Faraday rotation signal doubles



Magnetometry beyond the projection noise limit

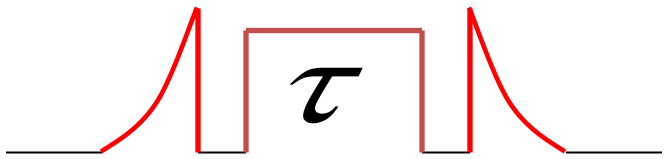
Entanglement *by QND measurement*

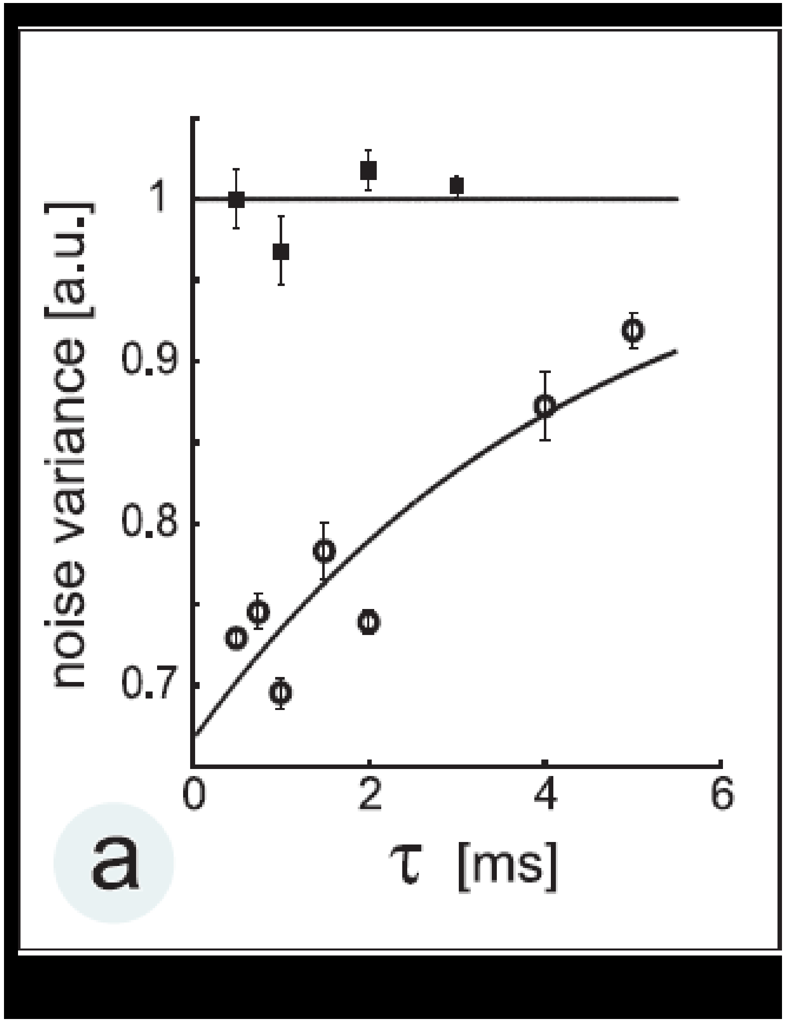


$$\delta\left(\hat{J}_{z1}+\hat{J}_{z2}\right)^2+\delta\left(\hat{J}_{y1}+\hat{J}_{y2}\right)^2=1.3\cdot J_x<2\cdot J_x$$

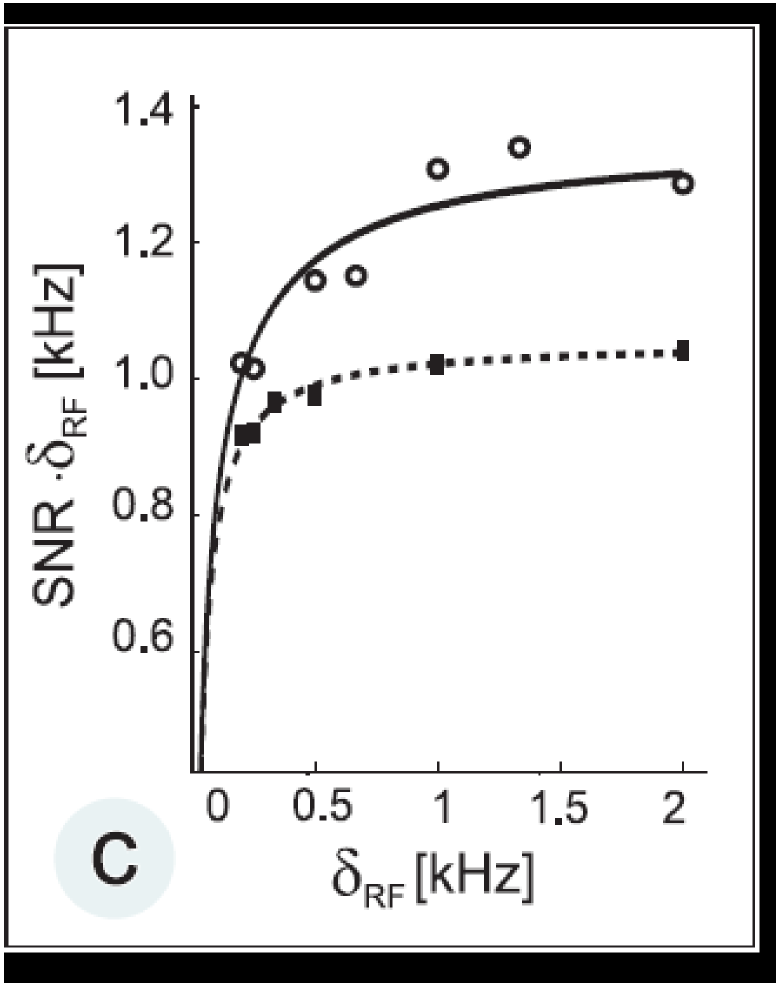
$$\delta\left(\hat{X}_1+\hat{X}_2\right)^2+\delta\left(\hat{P}_1+\hat{P}_2\right)^2=1.3<2$$

EPR entanglement

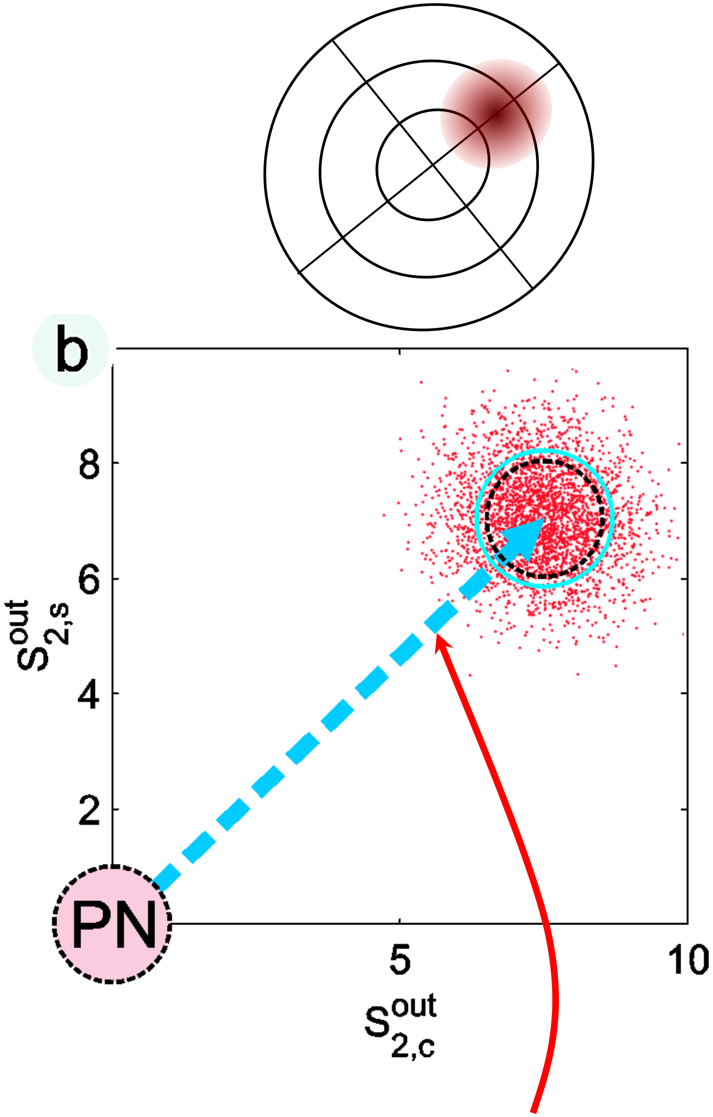




Signal/noise times bandwidth



Magnetic field sensitivity with 1.5*10^12 atoms



$$0.42 \cdot 10^{-15} T / \sqrt{Hz}$$

State-of-the-art cell magnetometer with 10^{16} K atoms
Lee at al, Appl. Phys.Lett. 2006

$$0.24 \cdot 10^{-15} T / \sqrt{Hz}$$

100-fold improvement in sensitivity per atom

$$B_{RF} = 36 \cdot 10^{-15} Tesla = 3.6 \cdot 10^{-10} G$$