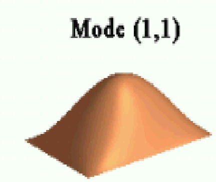
Quantum mechanical oscillator

$$X_{Lab}(t) = \mathrm{X}\sin(\omega t) + \mathrm{P}\cos(\omega t)$$

Dimensionless canonical variables

$$Var(X)Var(P) \geq 1/4 \qquad [X, P] = i$$

Oscillator energy $E = \hbar\omega(n + \frac{1}{2}) = \frac{1}{2}m\omega^2 x^2 = \frac{p^2}{2m}$



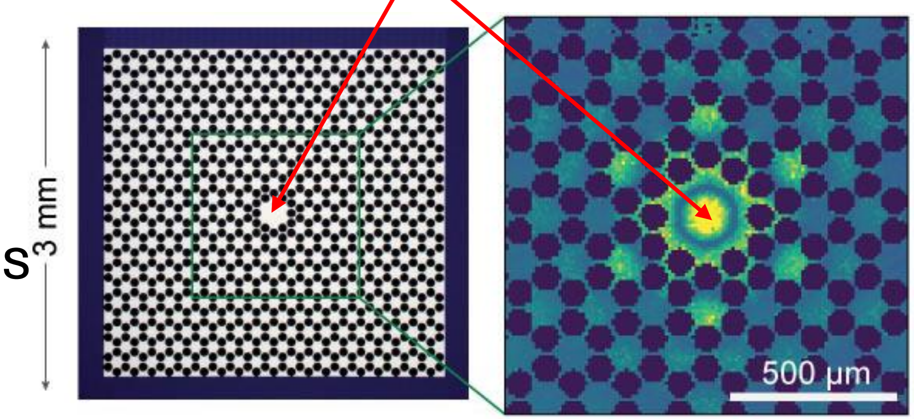
Zero-point fluctuations: $n = 0 \rightarrow E = \frac{1}{2}m\omega^2 x_{zpf}^2 = \frac{1}{2}\hbar\omega$

$$x_{zpf} = \sqrt{\frac{\hbar}{m\omega}} \sim 1\, fm$$

$m \approx 10ng;\ \omega \approx 2\pi \cdot 10^6 Hz$

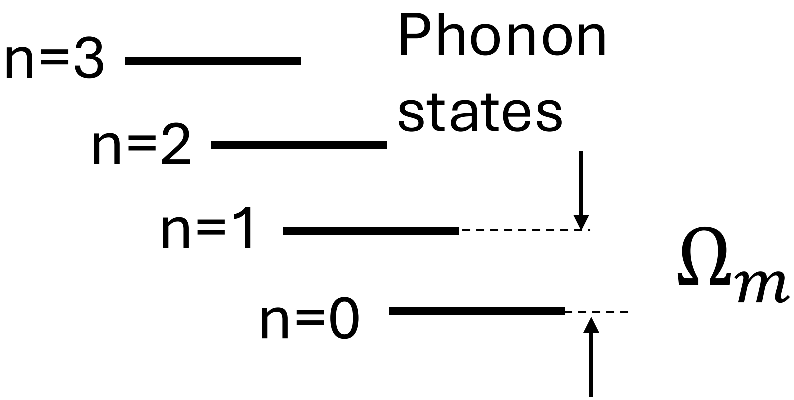
$$X = \frac{x}{x_{zpf}} \qquad P = \frac{p}{\hbar} x_{zpf}$$

Dimensionless canonical variables



Oscillator Hamiltonian

$$H = \frac{p^2}{2m} + \frac{m{\Omega_m}^2 x^2}{2} = \frac{\hbar}{2}\Omega_m(P^2 + X^2)$$



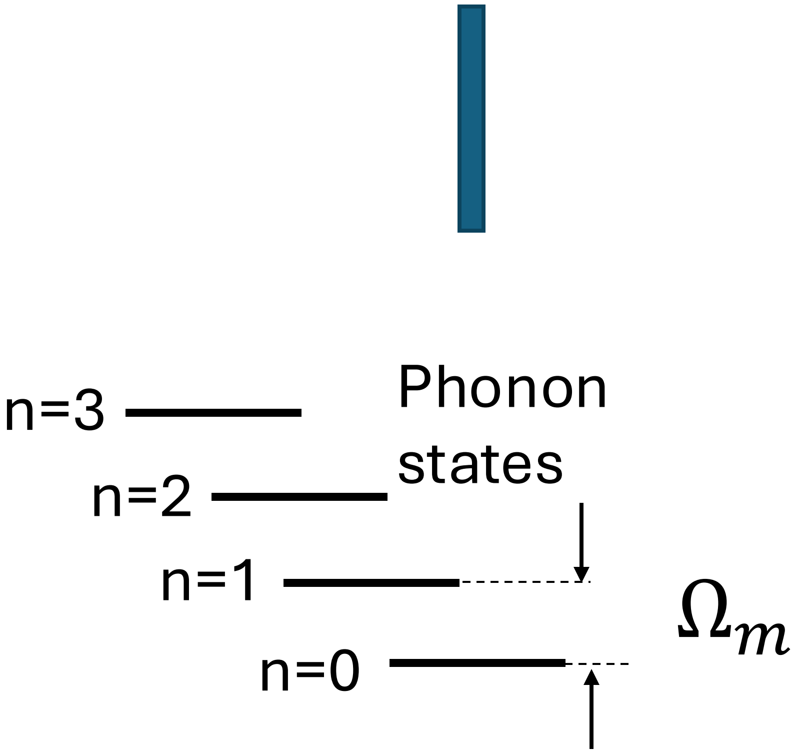
$$X = \frac{x}{x_{zpf}}, P = p x_{zpf}/\hbar$$

$$x_{zpf} = \sqrt{\frac{\hbar}{m\Omega_m}}$$

Zero-point fluctuations

Example: m = 10 ng, $\Omega_m = 2\pi\ 10^6 \ \rightarrow\ x_{zpf} = 10^{-15}$m

Thermal noise



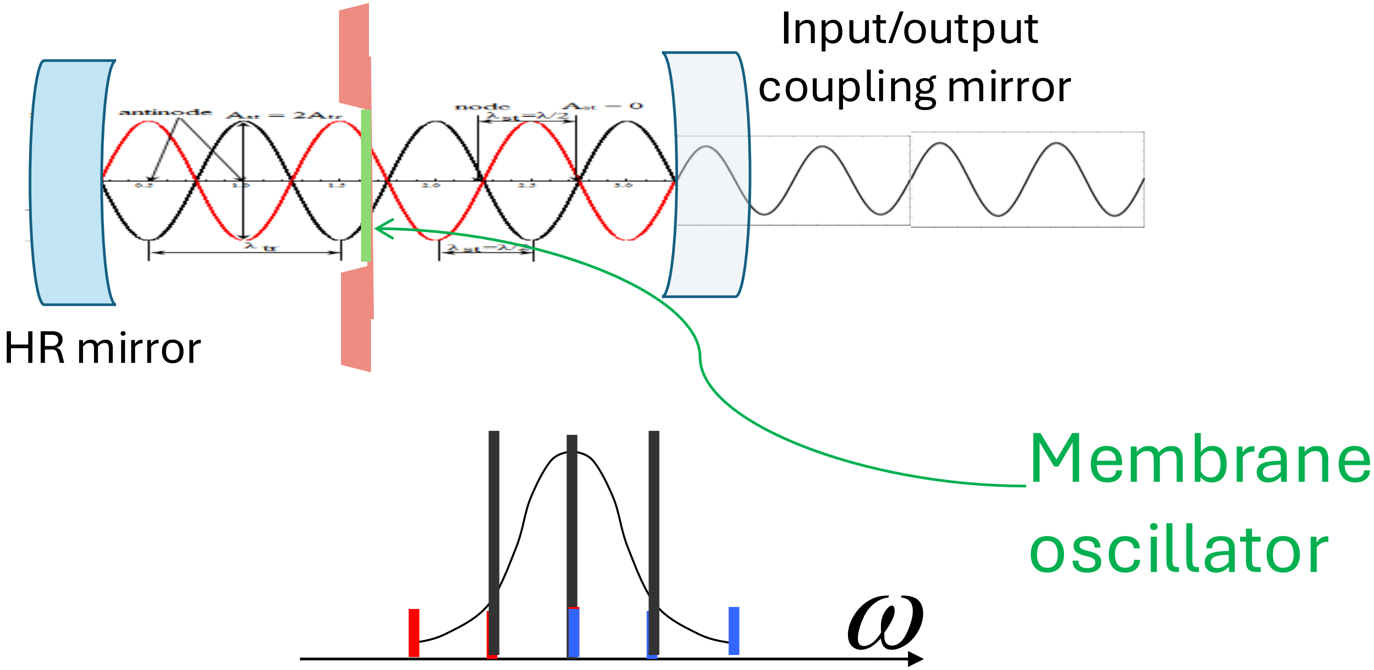
$$n_{th} = \frac{k_B T}{\hbar \Omega_m}$$

$$x_{th} = \sqrt{\frac{2 n_{th} \hbar}{m \Omega_m}}$$ Thermal fluctuations

Example: m = 10 ng, $\Omega_m = 2\pi\ 10^6$

$T = 4K \rightarrow\ x_{th} \approx 10^{-12}$m

Optomechanical photon-phonon interaction



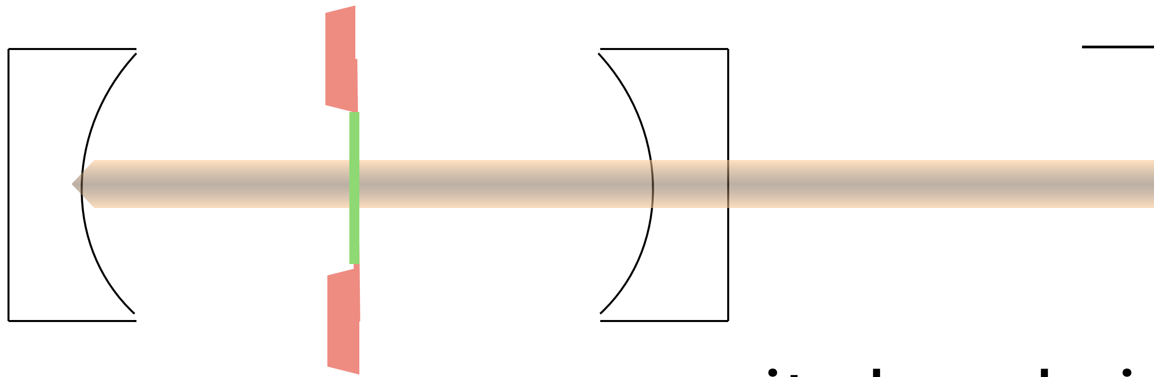
Optical cooling <-> state readout

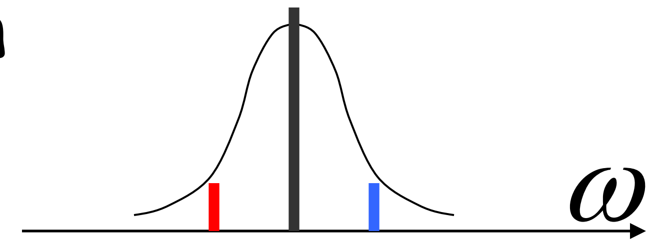
$$H = \chi_{Par}\hat{a}^{\dagger}\hat{b}^{\dagger} + \chi_{BS}\hat{a}^{\dagger}\hat{b} + h.c.$$

$$= gX_M X_L \qquad g = \chi_{Par} = \chi_{BS}$$

phoTon ⟷ **entangled** ⟷ phoNon

Optomechanical interaction (Quantum NonDemolition)





$$H = gX_M X_L$$

κ – cavity bandwidth

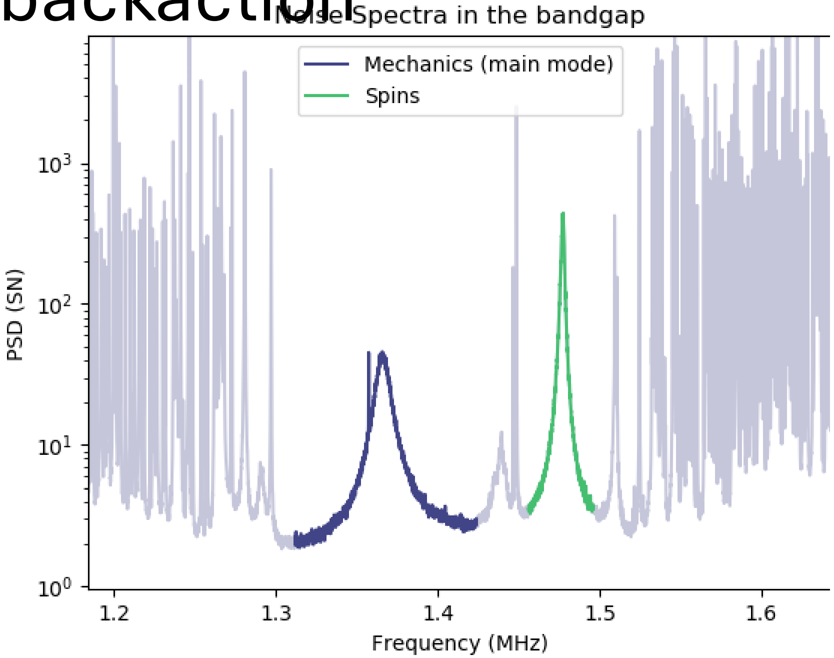
$$\hat{P}^M_{L,\mathrm{out}} = \hat{P}^M_{L,\mathrm{in}} - \sqrt{\Gamma_M}\hat{X}_M$$

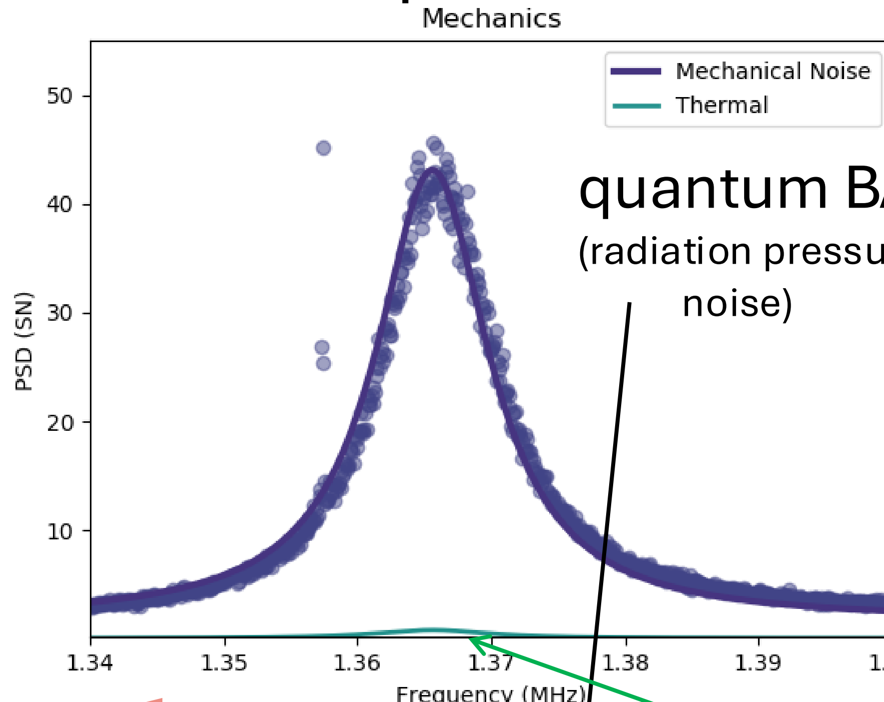
$$\Gamma_M = 2g^2/\kappa$$

$$g = \frac{\omega_{opt}}{L}\sqrt{\frac{\hbar n_{ph}}{m\Omega_m}}$$

Intracavity Photon number

Mechanics (membrane) spectrum and quantum backaction

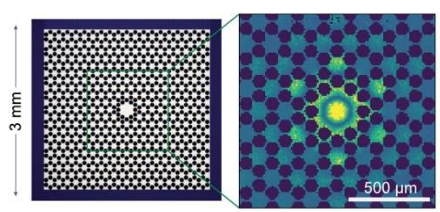


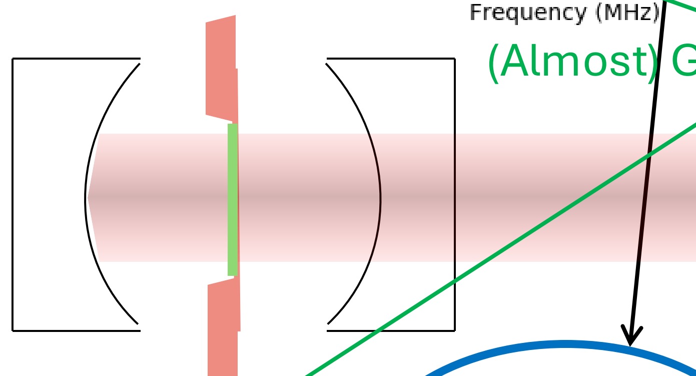


quantum BA

(radiation pressure noise)

(Almost) Ground state noise





Shot noise of light

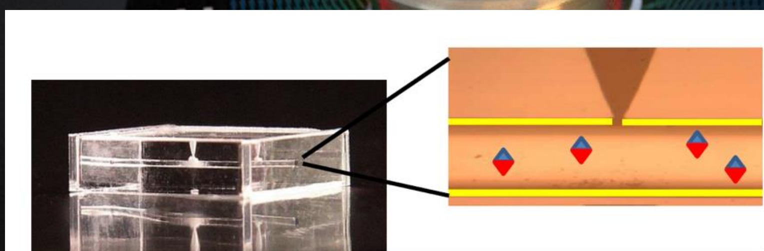
Back action of light

$$P_{L1,out} = -P_{L1,in} + force\ terms + \Gamma_M \chi_M X_{L1,in}$$

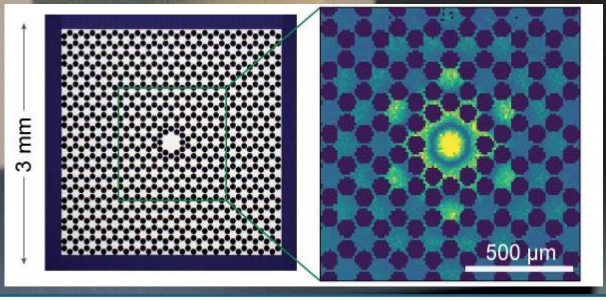
Quantum back action free measurement of motion

for microscopic oscillator

C. Møller et al, Nature (2017)



spin oscillator



Mechanical oscillator

Probe phase	Spin readout	Mechanics readout	Measurement backaction on spin	Measurement backaction on mechanics

$$P_{Lout} = P_{L,in} + P_S + P_M + \cancel{\Gamma_S \chi_S X_{L,in}} + \cancel{\Gamma_M \chi_M X_{L,in}}$$

Image credit
Bastian Leonhardt Strube and Mads Vadsho

Matching quantum back actions for hybrid mechanical – spin system

Matching ”masses” ⟷ Matching cooperativities

$$\kappa^2 \Gamma_S = \frac{4g^2}{\Gamma_c}$$

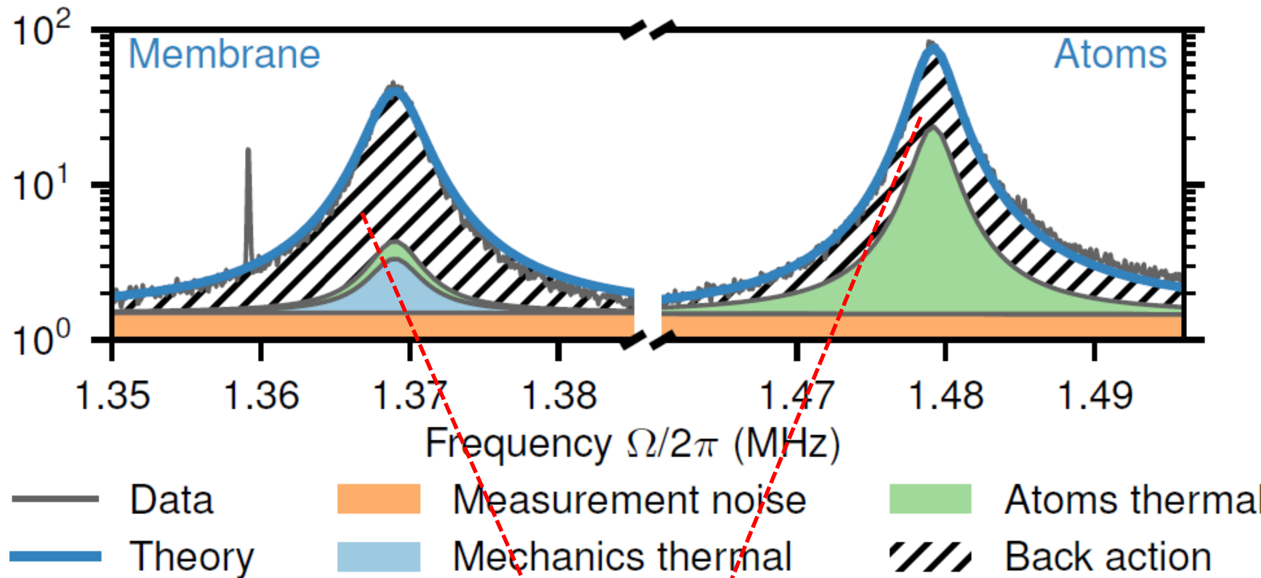
$\kappa^2 \leftrightarrow$ optical depth

$$g = \frac{\omega_{opt}}{L}\sqrt{\frac{\hbar n_{ph}}{m\Omega_m}}$$

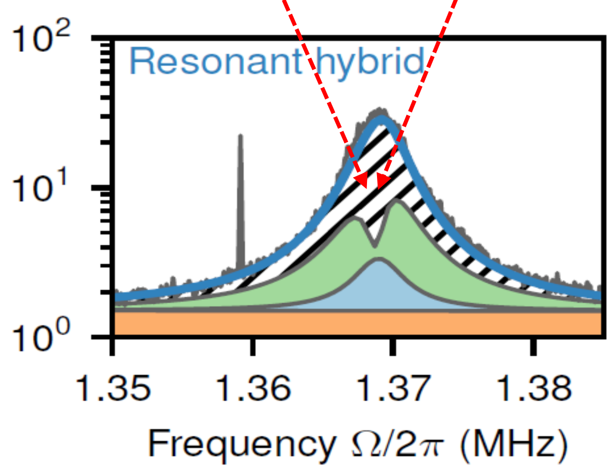
$$H_{spin} = \frac{\kappa}{\tau_p} X_{spin} x_{light}$$

$$H_{mech} = g x_{Mech} x_{light}$$

Cancellation of Quantum backaction noise in negative mass reference frame



C. Møller et al.
Nature 2017



-6 dB reduction of QBA

$$\Gamma_M \chi_M = -\Gamma_S \chi_S$$

$$P_{Lout} = P_{L,in} + force\ terms + \cancel{\Gamma_M \chi_M X_{L1,in} + \Gamma_S \chi_S X_{L2,in}}$$

Generation of entangled state of mechanical and spin oscillators

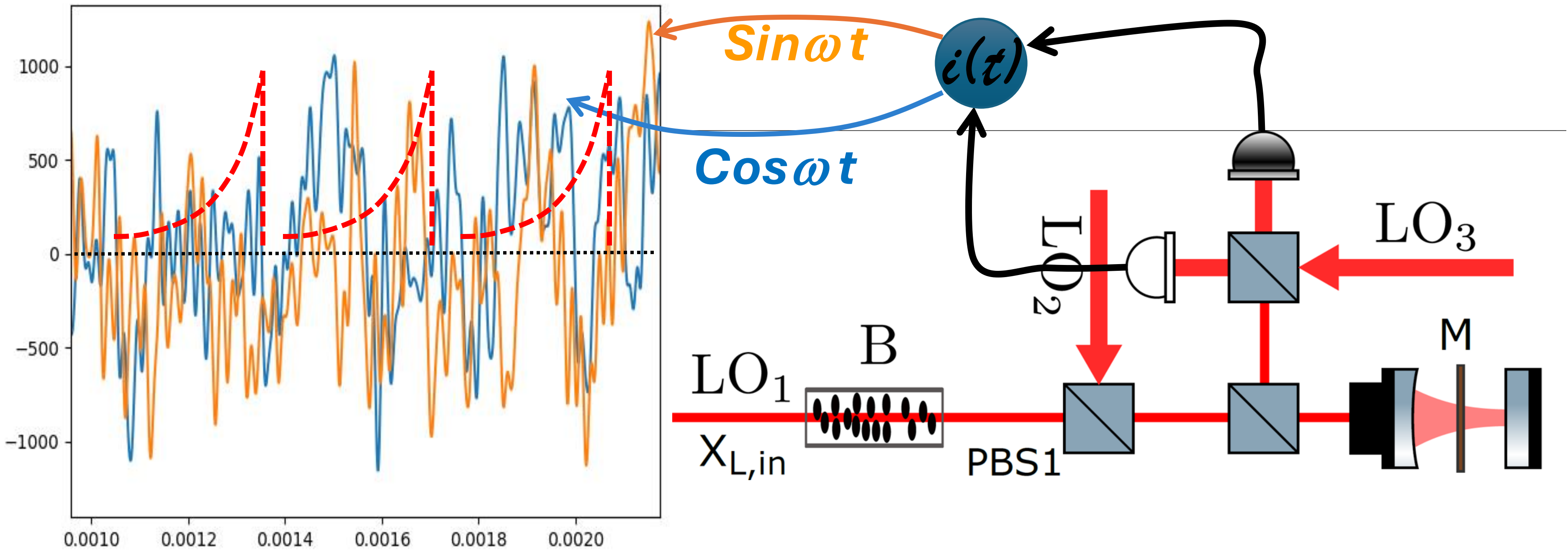
$$X(dt)_{X0} = X - X_0 + (P + P_0)dt$$

$$Var(X - X_0) + Var(P + P_0) \rightarrow 0$$

MSc degree fromState University of Maringa, Brazil
PhD at the Niels Bohr Institute

R. Thomas et al, **Nature Phys.** 17, 228–233(**2021**)

Entanglement generation by continuous measurement (Wiener filtering)



Conditional variance of a quantum state

i(t) i(t')

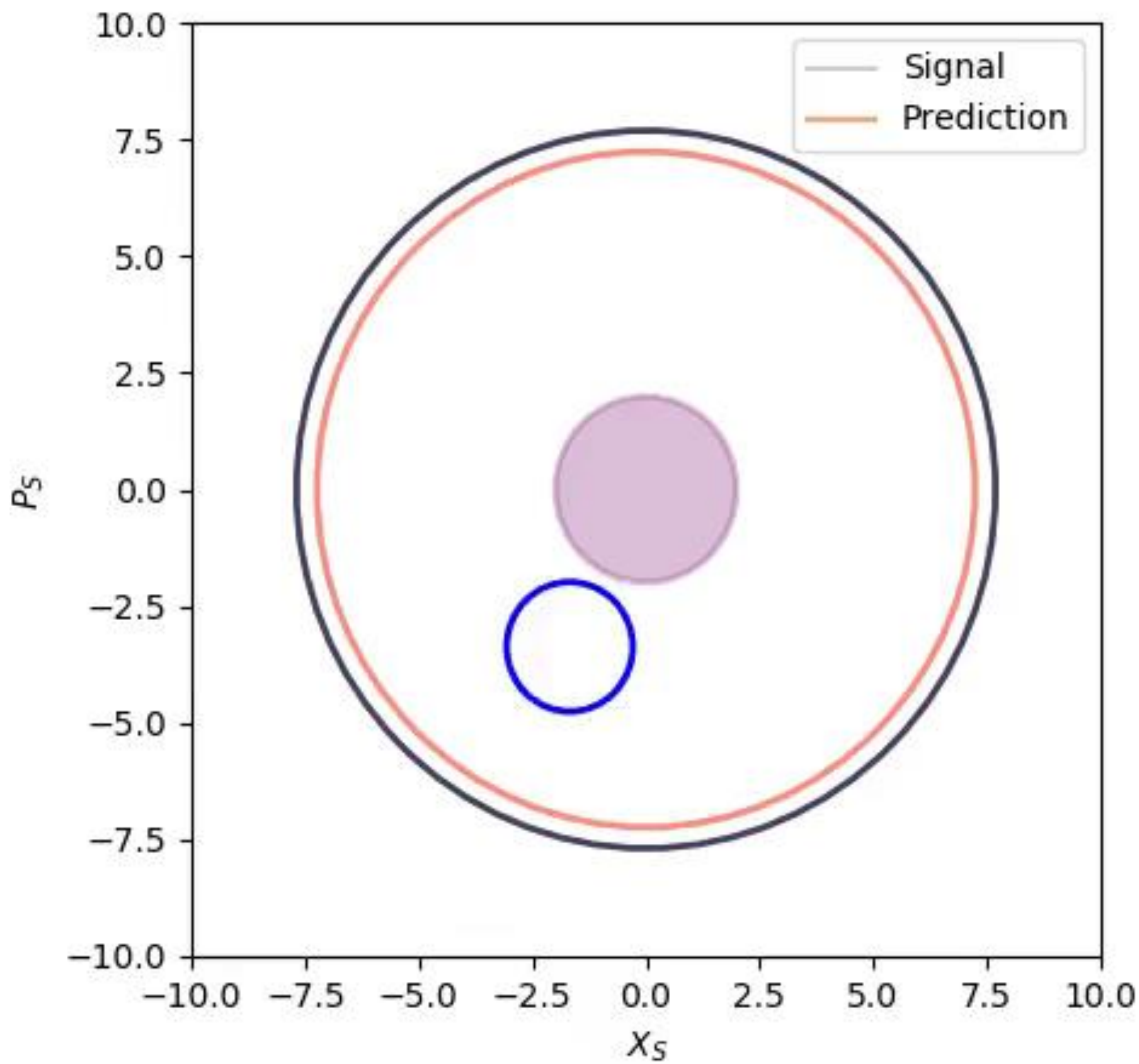
$$V_c = \langle R(t)^2 \rangle = \underbrace{\langle x(t)^2 \rangle}_{V_u} - \langle \vec{x}(t)^2 \rangle$$

$$x(t) = \underbrace{\int_{-\infty}^{t} \mathrm{d}t' K(t-t') y(t')}_{\vec{x}(t)} + R(t)$$

noise

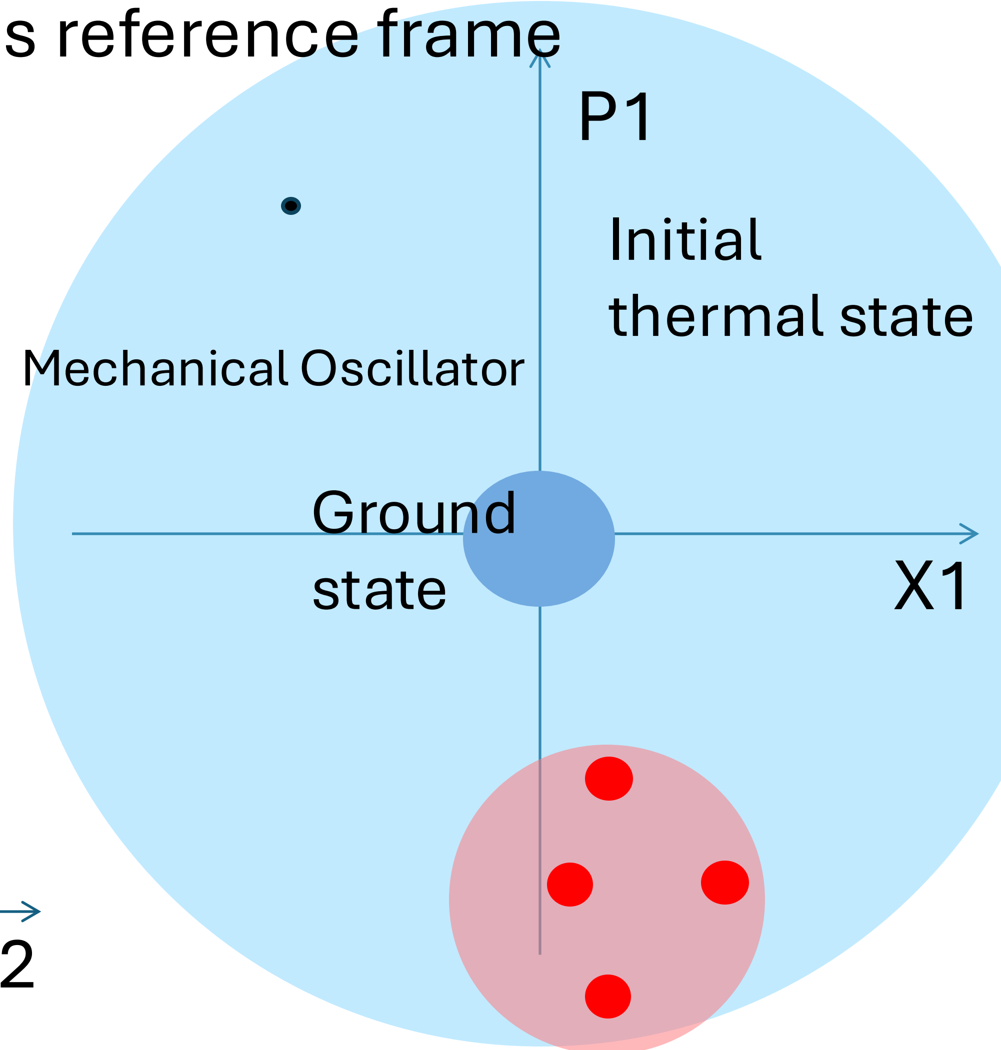
$$X(t) - X_0(t) = [X(0) - X_0(0)]\cos(\omega t) + [P(0) + P_0(0)]\sin(\omega t)$$

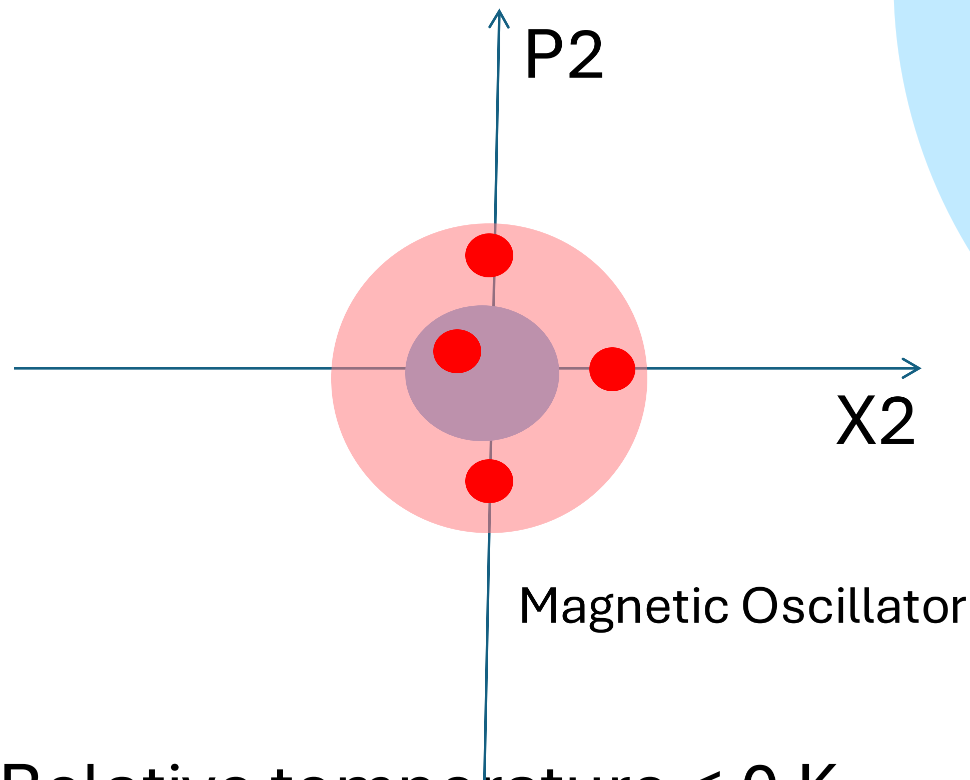
$$V_c = \langle R(t)^2 \rangle = \underbrace{\langle x(t)^2 \rangle}_{V_u} - \langle \vec{x}(t)^2 \rangle$$



Trajectory in a negative mass reference frame

Entangled state of two oscillators = trajectories in X,P space are correlated better than SQL





Experiment: Time without a jump at 8K

$$\frac{\hbar Q}{k_B T} \approx 0.3\text{msec}$$

Relative temperature < 0 K

Quantum Sensing in Health Applications

Copenhagen Center for Biomedical Quantum Sensing

Eugene Polzik

Niels Bohr Institute

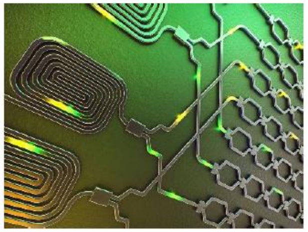
novo nordisk fonden

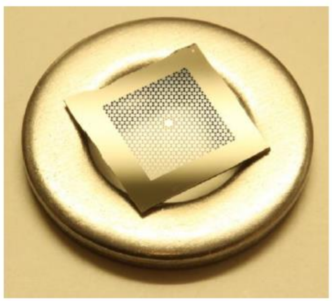
Image credit Bastian Leonhardt Strube and Mads Vadsholt

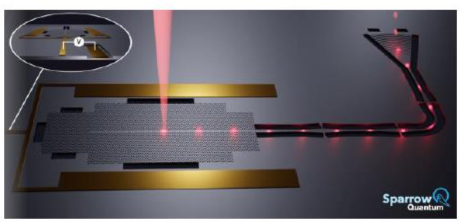
Quantum Sensing

Quantum Technology

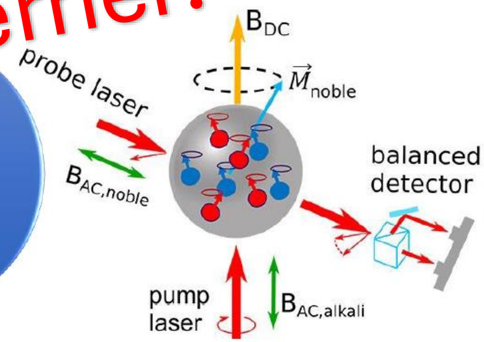
The more qubits, the merrier!







Quantum Sensors



Quantum Technology

Quantum Cryptography

~1 qubit per pulse

Quantum Computing

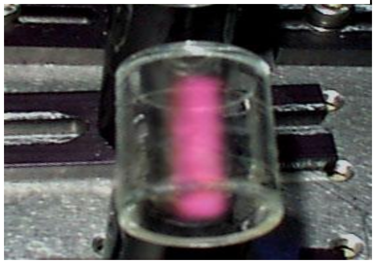
10-100 qubits

Sensing
of electro-magnetic fields

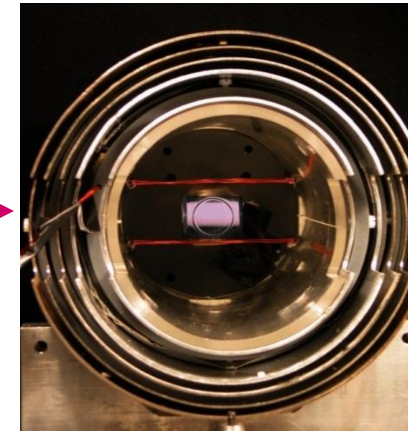
Standard Quantum Limit similar to Heisenberg microscope

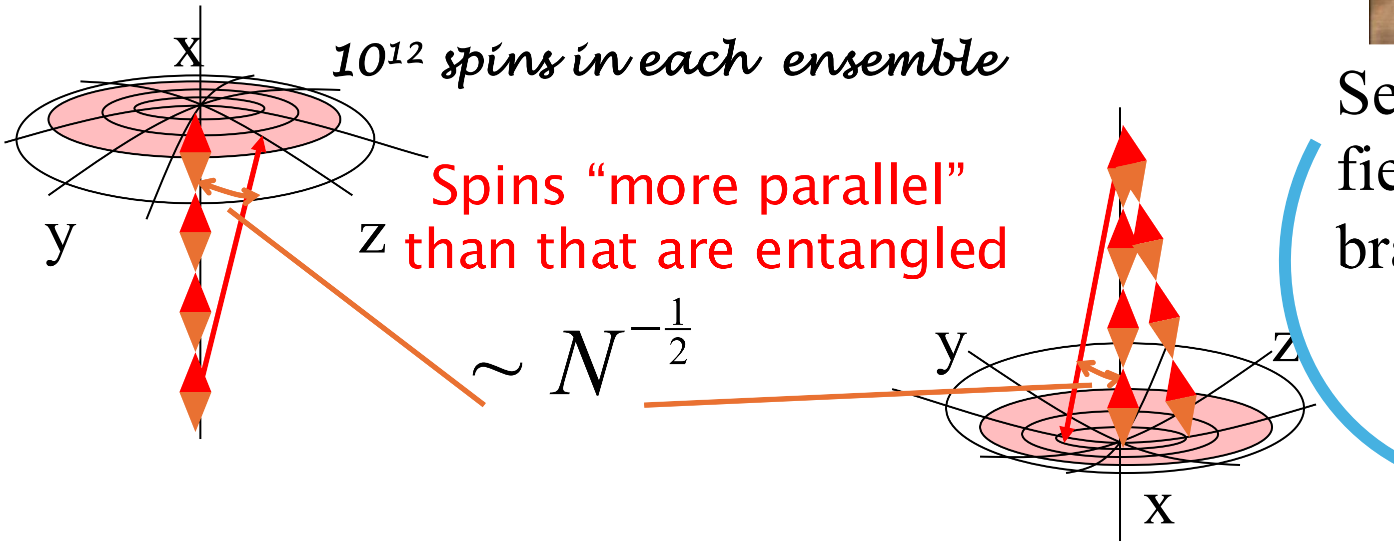
balance between Imprecision and Backaction

Sensing beyond SQL
enabled by entanglement



Entanglement assisted magnetometry





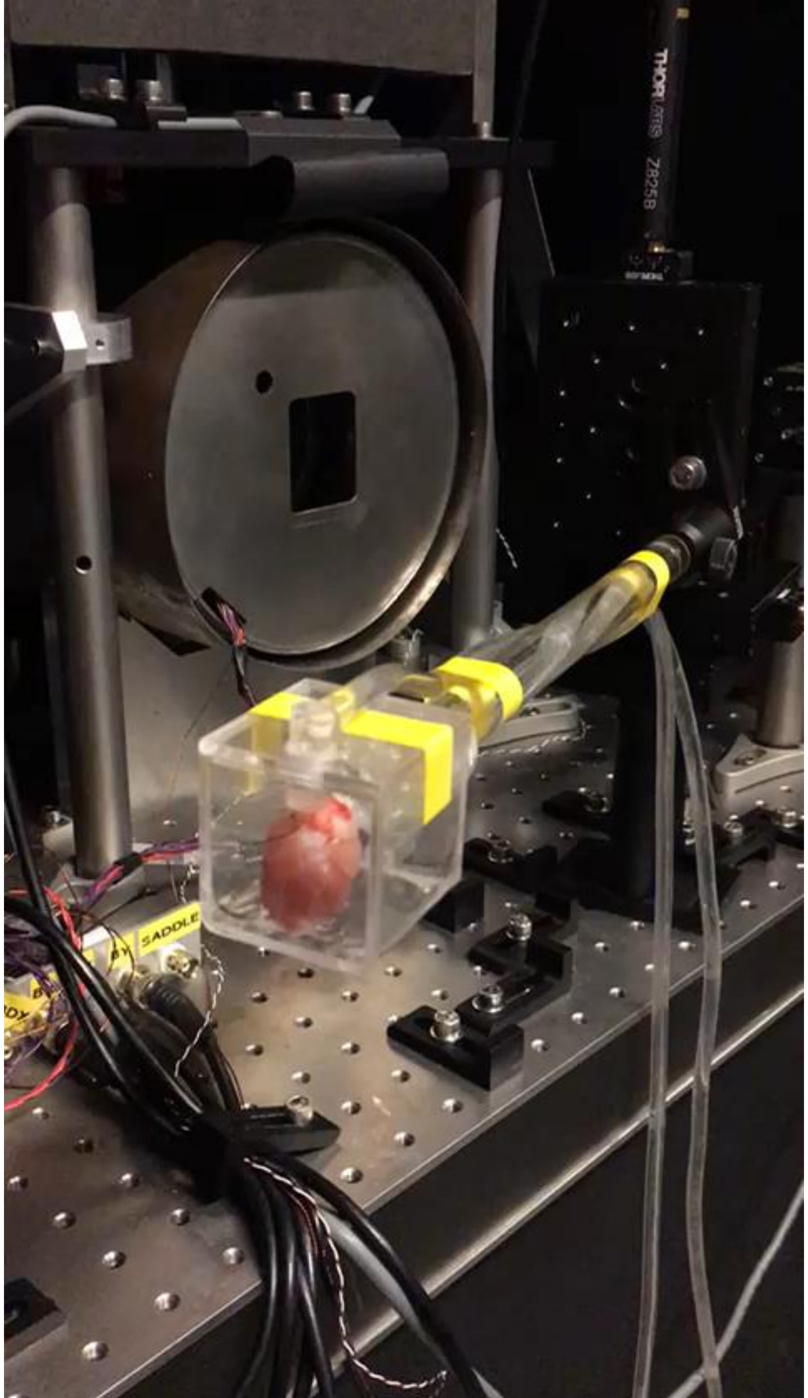
Sensor for magnetic field from brain, heart, nerve…

Minimal uncertainties for uncorrelated spins $\partial J_{z1,2}^{2} = \partial J_{y1,2}^{2} = \frac{1}{2} J_{x} \;\rightarrow\; \partial J_{y,z} \sim \sqrt{J_x} \sim \sqrt{N}$

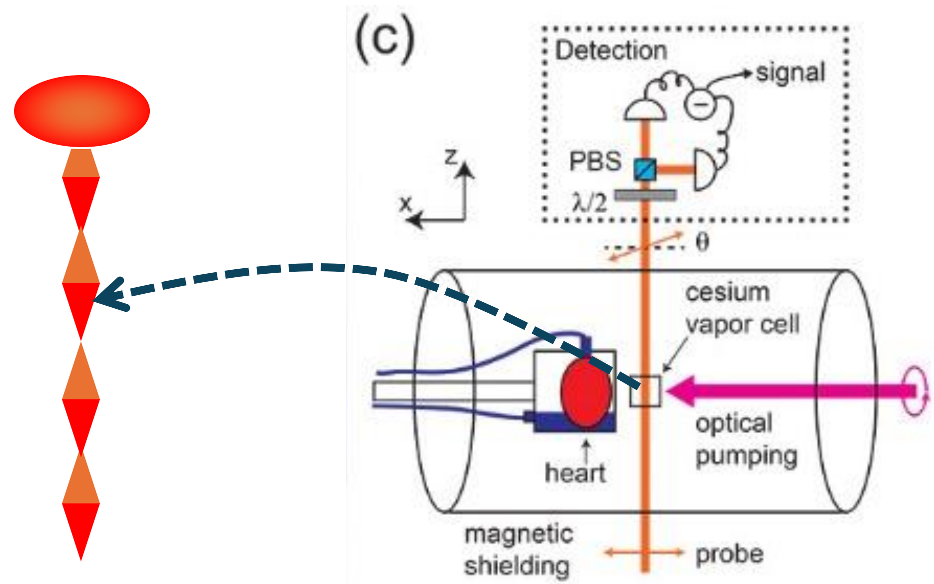
We measure magnetic fields 10^{11} times weaker than Earth magnetic field

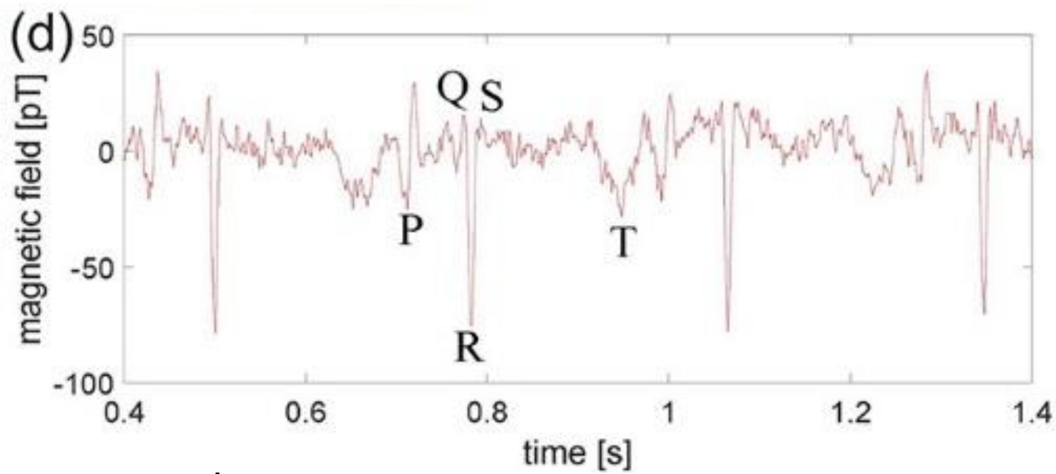
QND measurement. Entangled for 2 millisec. ***Nature*****, 413**, 400 (2001)

Steady-state entanglement by dissipation+measurement. ***PRL*****, 107**, 080503 (2011).



Atomic Spin magnetometry for **Noninvasive cardiac diagnostic**

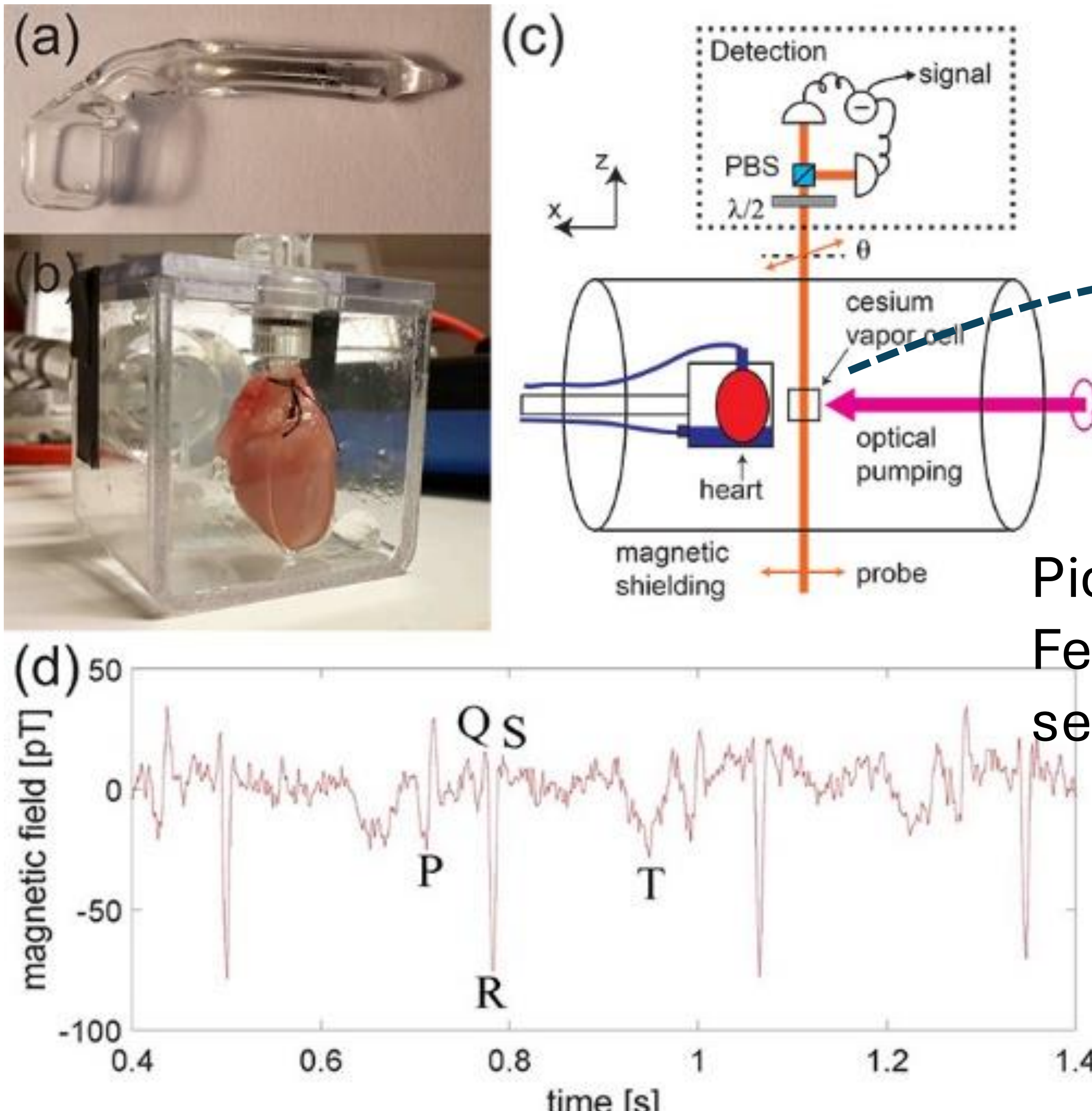


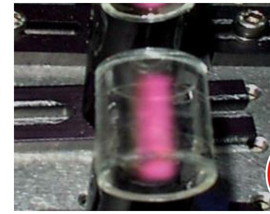


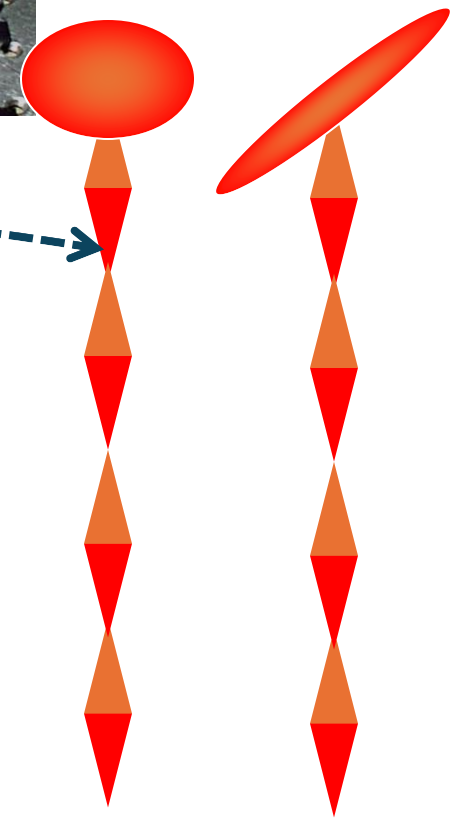
Jensen et al.
Scientific Reports (2018)

NBI collaboration with
Rigshospital, Copenhagen

Atomic Spin magnetometry for **Noninvasive cardiac diagnostics**







Picotesla to Femtotesla sensitivity

NBI collaboration with Rigshospital, Copenhagen

Entangled state of 10^{9}- 10^{11} atoms

Only very special types of such states survive

High symmetry helps

- single mode squeezed states

- two mode squeezed states = EPR entangled

Julsgaard et al ***Nature*** 2001; Sherson et al ***Nature*** 2006;
Thomas et al **Nature Physics** 2021

- symmetric collective single excitations (Fock states)

Dideriksen et al **Nature Comm** 2021

Suppressing quantum noise in sensing

Quantum

probe $P_{L1,in}, X_{L1,in}$

Sensor

Detection $P_{L1,out}$

Shot noise of probe

Measurement rate

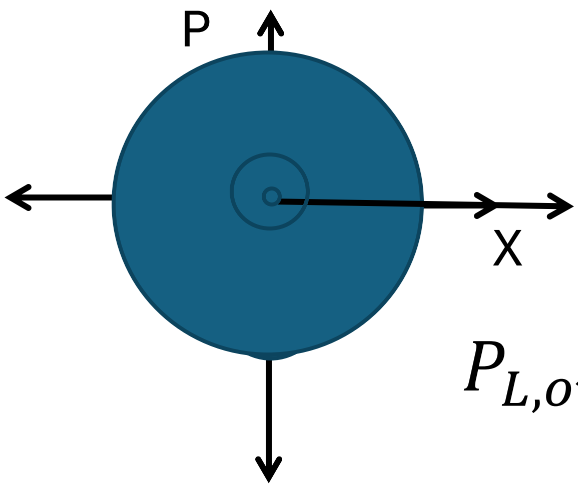
Back action of measurement

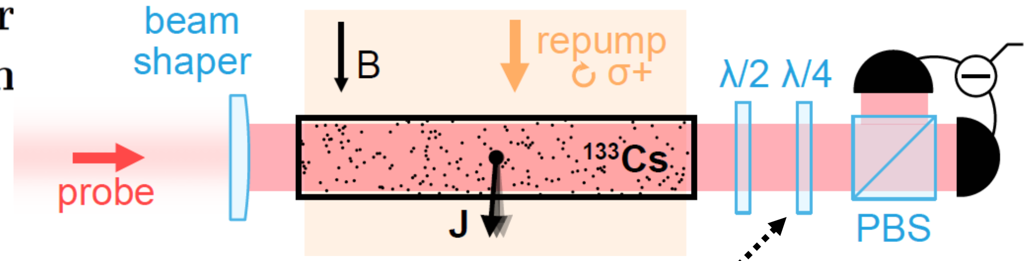
$$P_{L1,out} = -\cancel{P_{L1,in}} + sensor\ q.noise + \Gamma\, \cancel{\chi X_{L1,in}} + signal$$

Squeezed Light probe

Quantum engineering

Squeezed light from an oscillator measured at the rate of oscillation





Shot noise of light

Measurement rate

Back action of light

$$P_{L,out} = -P_{L,in} + signal + \Gamma\chi\, X_{L,in}$$

$$\hat{Q}_{\mathrm{L}}^{\phi} = \sin(\phi)\hat{X}_{\mathrm{L}}^{\mathrm{out}} + \cos(\phi)\hat{P}_{\mathrm{L}}^{\mathrm{out}}$$

Cross-correlations

Back action of light on oscillator

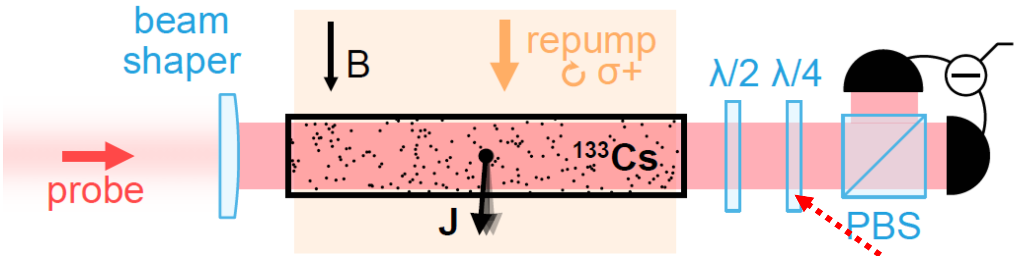
$$S_\phi[\Omega] = 1/4 + (\eta\Gamma/2)\,\mathrm{Re}\,(\chi[\Omega])\sin(2\phi) + \eta\Gamma(\Gamma+\gamma_{\mathrm{th}})|\chi[\Omega]|^2\cos(\phi)^2$$

Shot noise of light

Γ - measurement rate ∝ **ensemble optical depth** (cooperativity)

C. Bærentsen et al, ***Nature Comm.*** **15**, 4146 (2024)

12 dB squeezed light for Sensing generated by Atomic gas



Destructive interference

Spin state readout

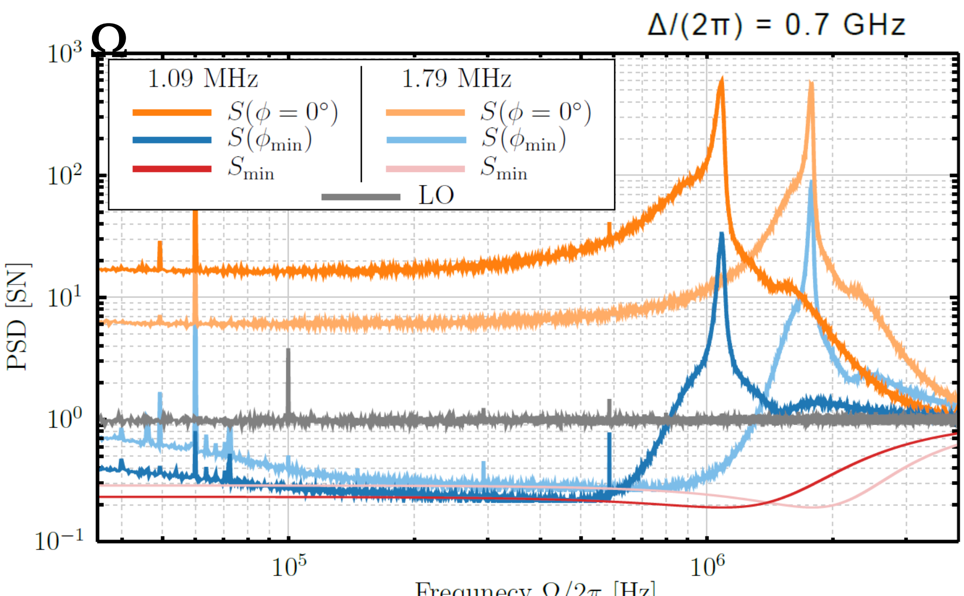
Back action of light on oscillator

$$S_\phi[\Omega] = 1/4 + (\eta\Gamma/2)\,\mathrm{Re}\,(\chi[\Omega])\sin(2\phi) + \eta\Gamma(\Gamma+\gamma_{\mathrm{th}})|\chi[\Omega]|^2\cos(\phi)^2$$

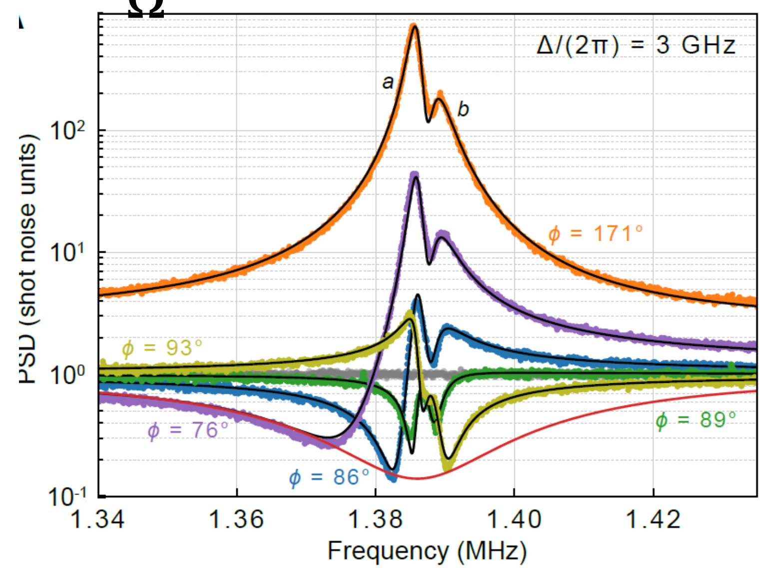
Can be negative

Fast measurement $\Gamma > \Omega$

Slow measurement $\Gamma < \Omega$



Broadband squeezing

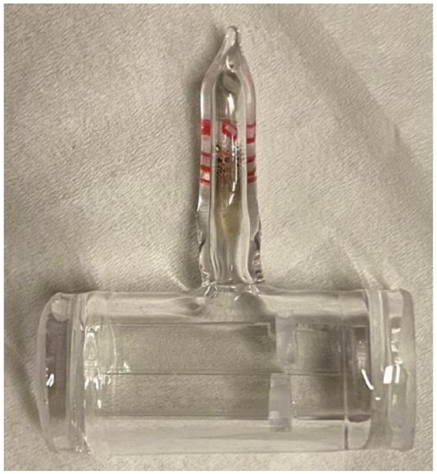


12dB squeezing
from RT atomic gas

C. Bærentsen et al, ***Nature Comm.* 15**, 4146 (2024)

Perspective: combining squeezed probe with entangled spins for Magnetic Sensing

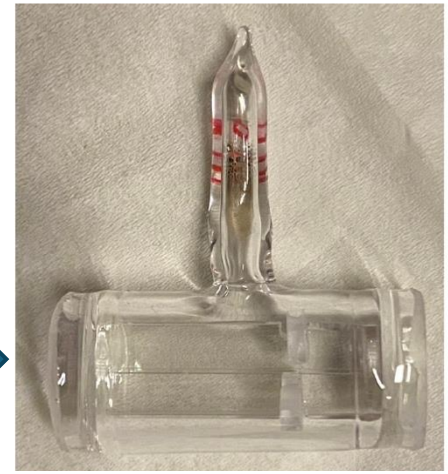
Laser



Cell 1: squeezer

Squeezed

probe



Cell 2: sensor

Detection

Shot noise of light

Measurement rate

Back action of light

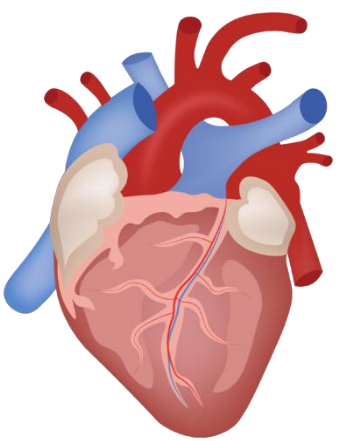
$$P_{L1,out} = -\cancel{P_{L1,in}} + signal + \Gamma \chi \, X_{L1,in}$$

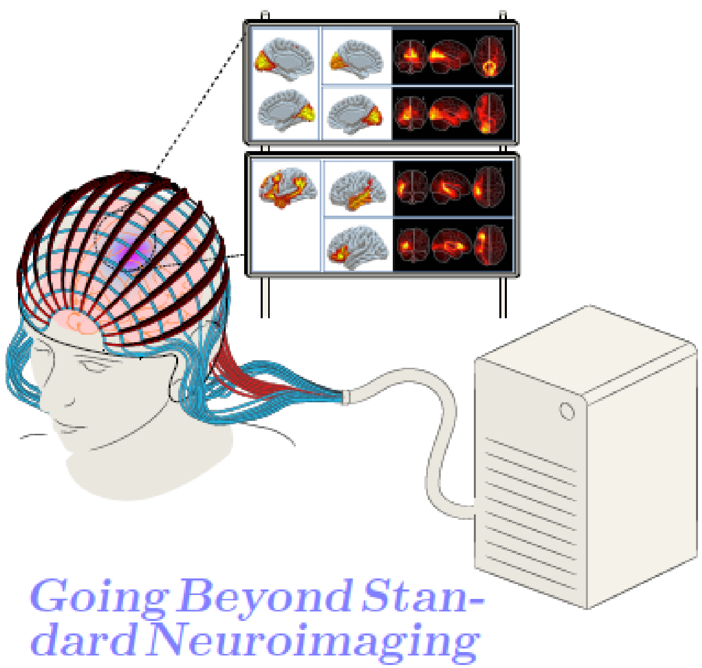
Squeezed Light probe

Improved spacial resolution with smaller sensor

Atomic Magnetic Induction Tomography

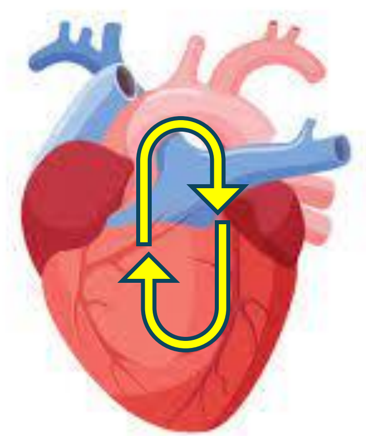
Distant noninvasive detection of weakly conducting objects



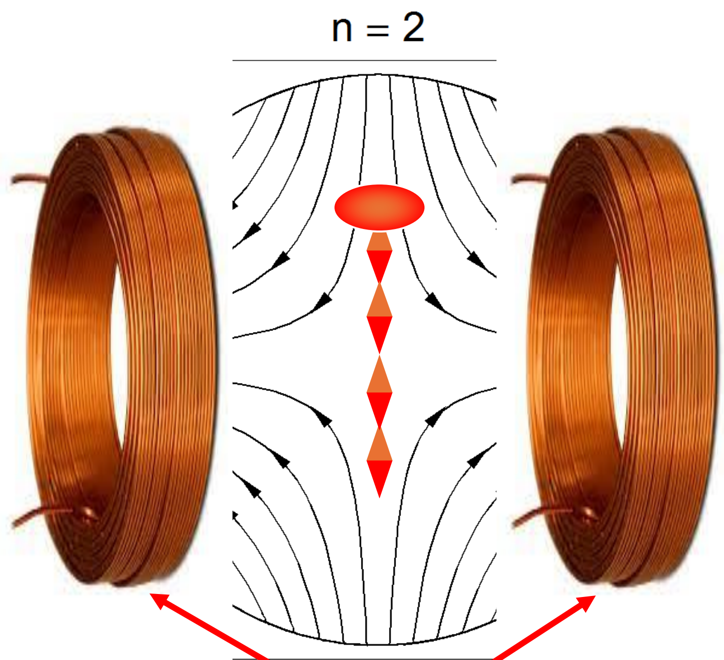


Atomic Magnetic Induction Tomography (Atomic MIT)

Distant noninvasive detection of weakly conducting objects



Induced currents



Anti-Helmholz coils

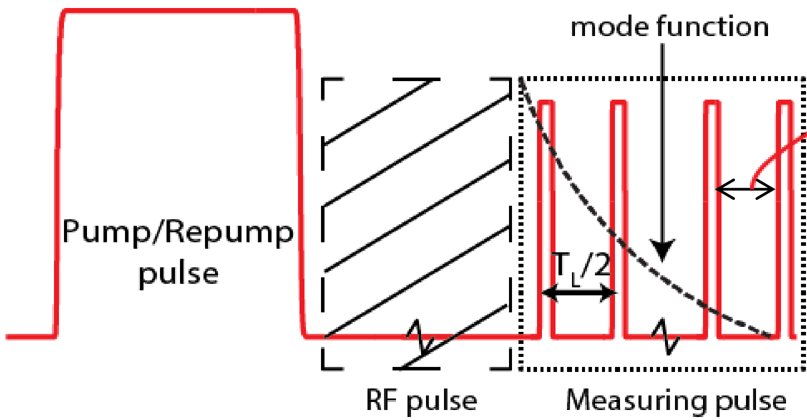
Key advantage for quantum enhancement:
Signal grows as square of field frequency

Squeezed state of oscillator generated by Stroboscopic Quantum Nondemolition measurement

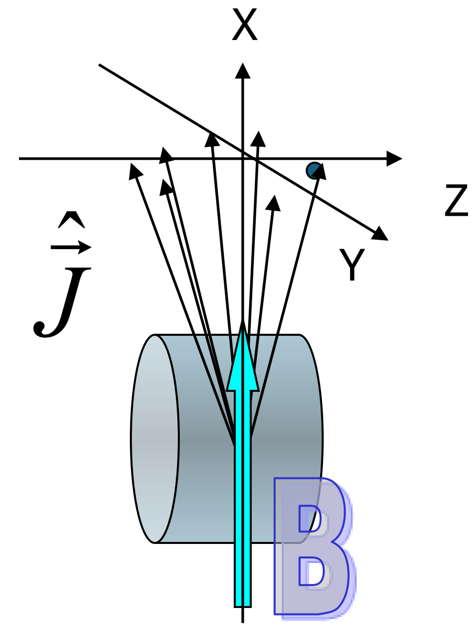
G. Vasilakis et al, ***Nature Phys***., *April 2015*.

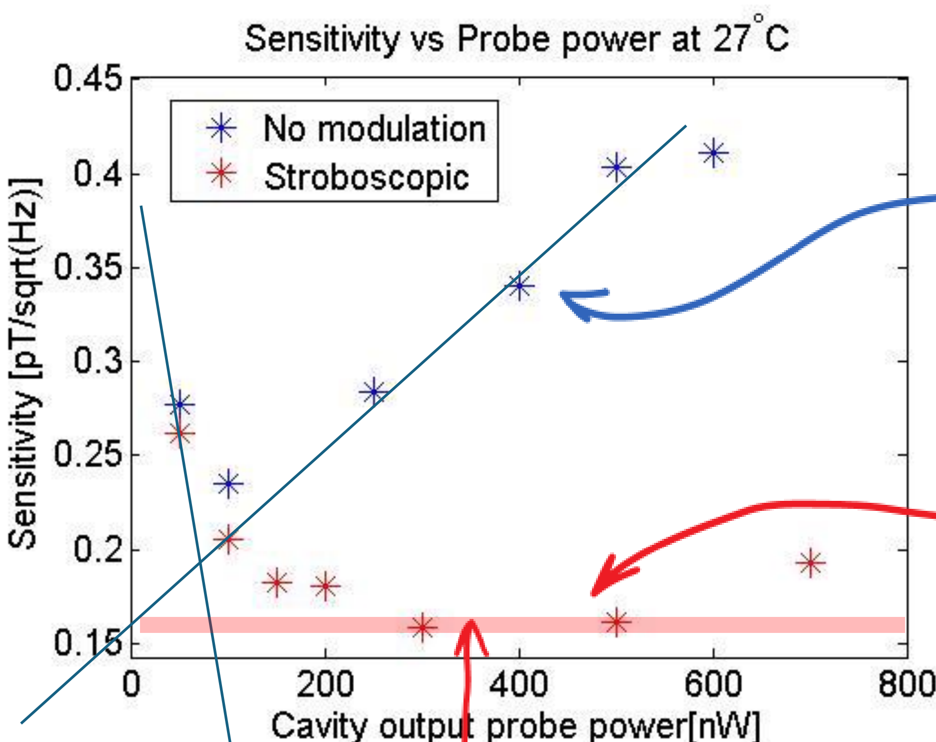
Proposal:
Braginsky, Vorontsov, Khalili et al, 1970s

Primer: Back action evading stroboscopic measurement on an oscilla

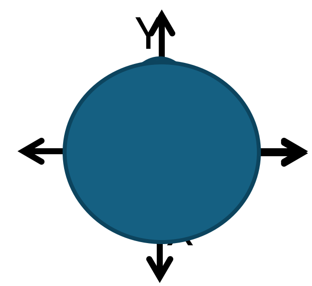


$\Delta t = \frac{1}{2\Omega}$

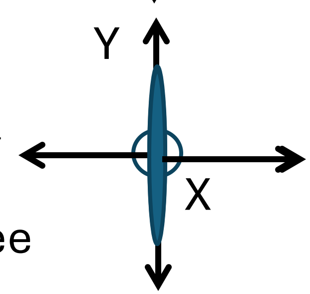




Continuous measurement leads to growing backaction



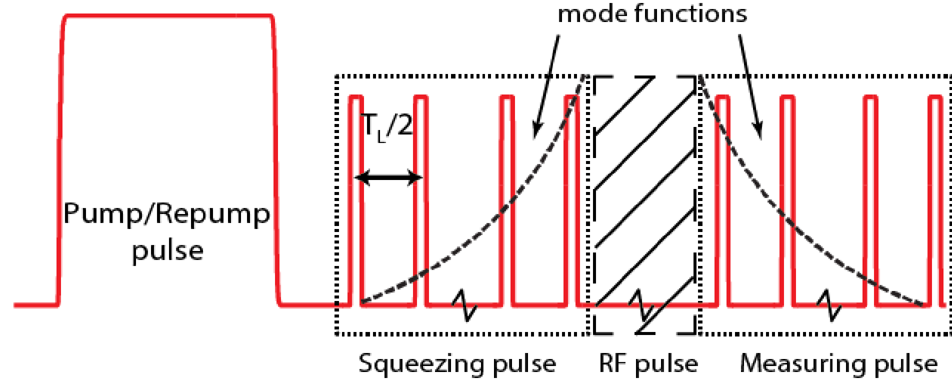
Stroboscopic measurement at 2Ω is backaction free

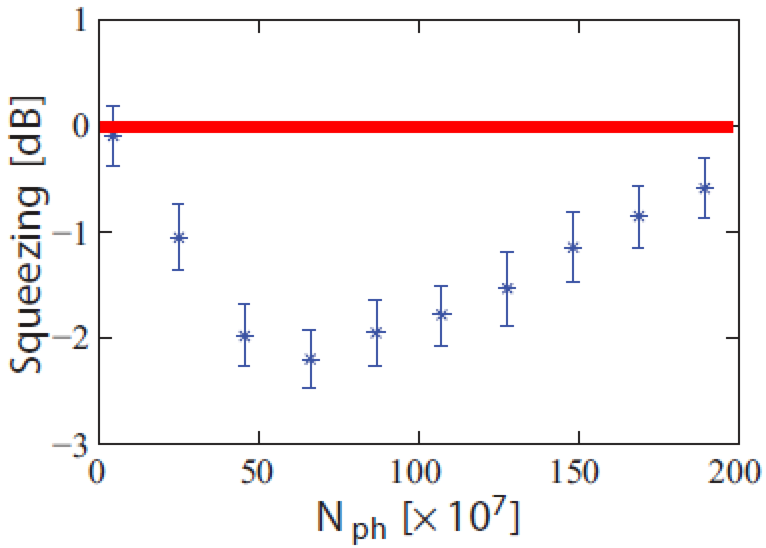


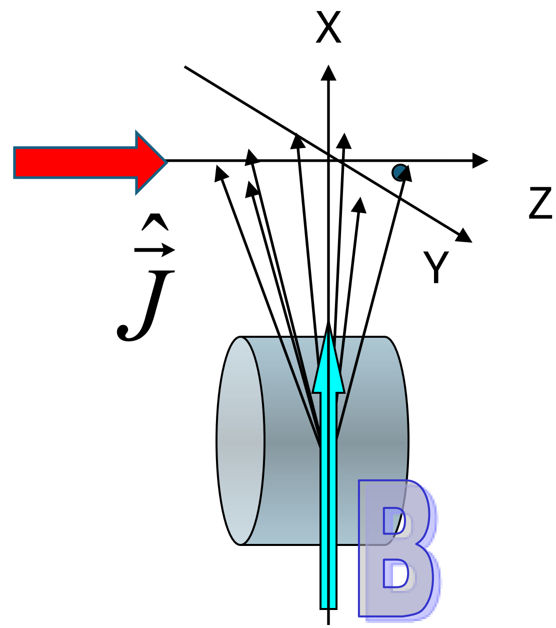
Ground state – projection noise

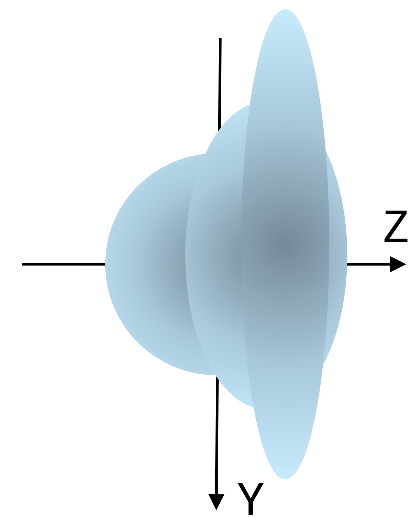
Nature Physics 11, 389–392 (2015)

Squeezed state of an oscillator consisting of 10^8 atoms





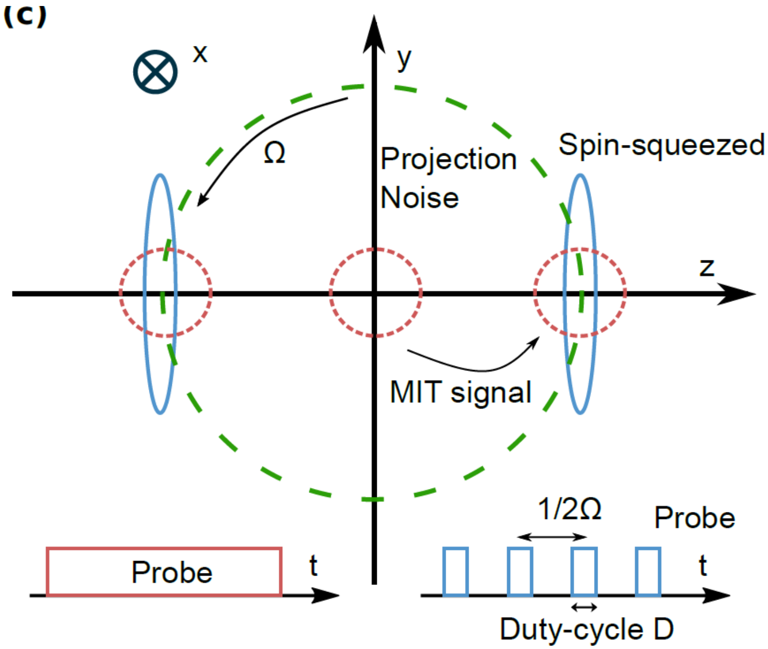


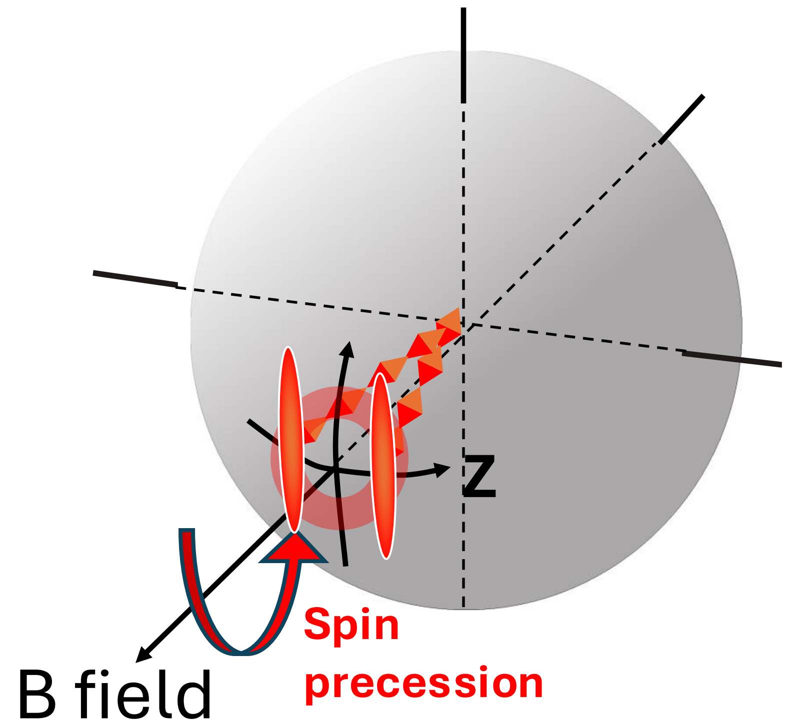


SQUEEZING BY REPEATED QND

G. Vasilakis et al, Nature Physics 11, 389–392 (2015)

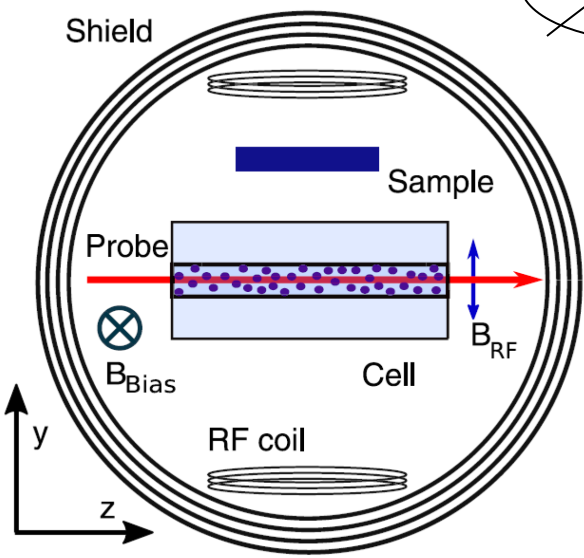
Entanglement assisted MIT

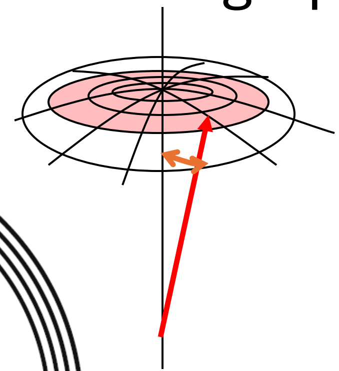


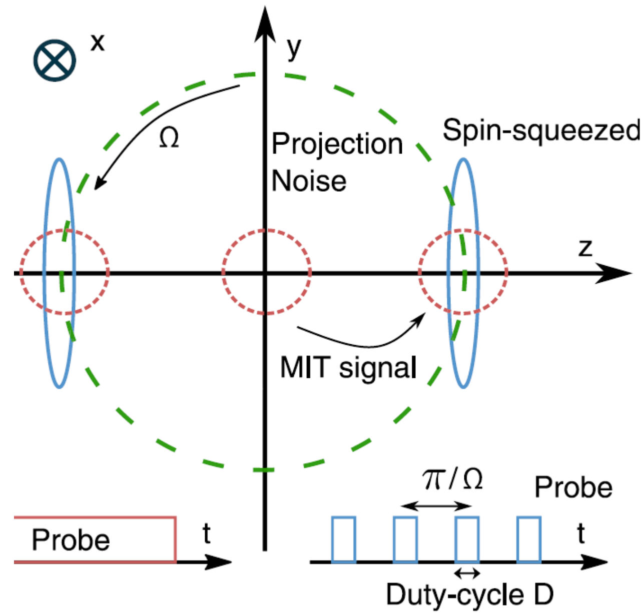


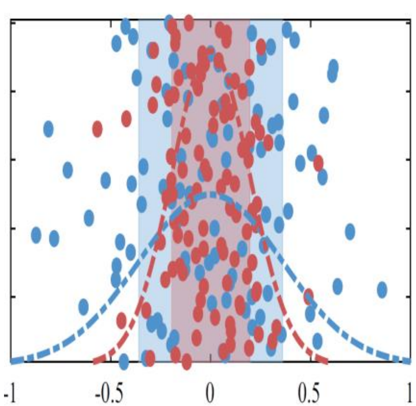
Stroboscopic projection
measurement on Z axis
collapses the spin state into
a squeezed state

Magnetic Induction Tomography with Spin Squeezed Sensor Atoms









~~Shot noise of light~~

Measurement rate

~~Back action of light~~

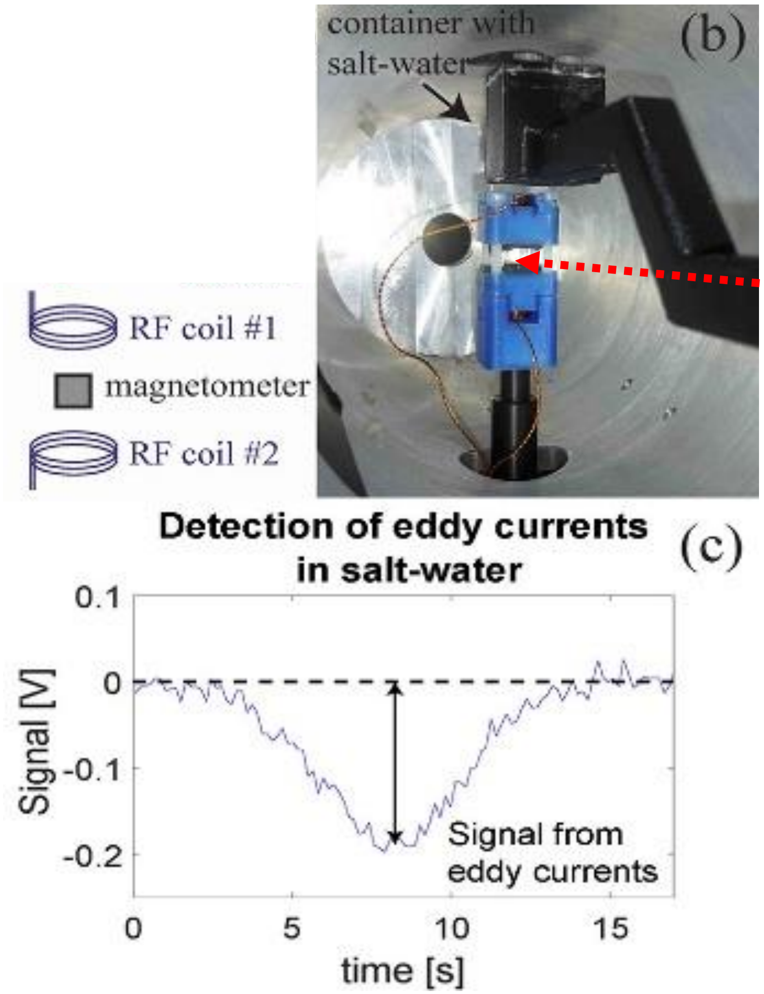
$$P_{L1,out} = \cancel{-P_{L1,in}} + signal + \Gamma \cancel{\chi X_{L1,in}}$$

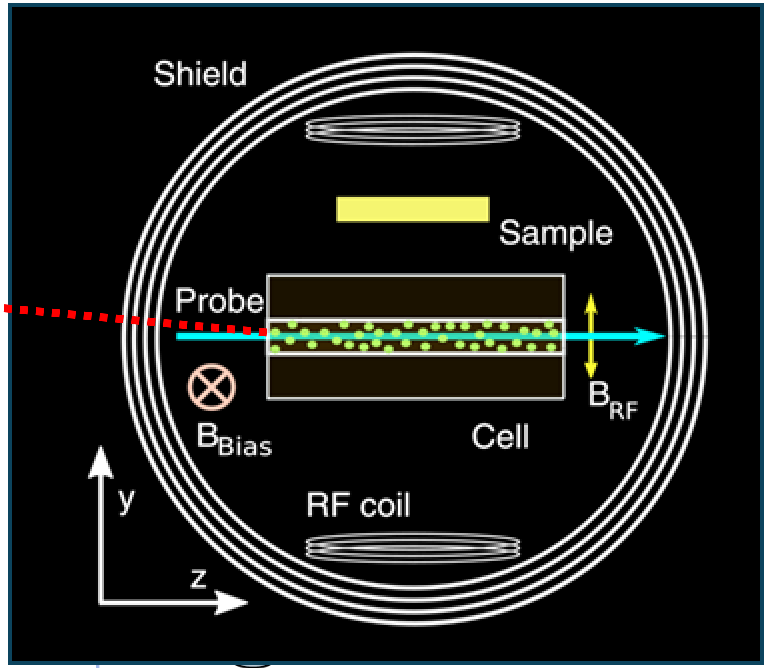
Future option: Squeezed Light probe

PHYSICAL REVIEW LETTERS **130,** 203602 (2023)

Editors' Suggestion **Featured in Physics**

Magnetic Induction Tomography with Entangled Atoms
Towards brain/heart diagnostics via conductivity measurement

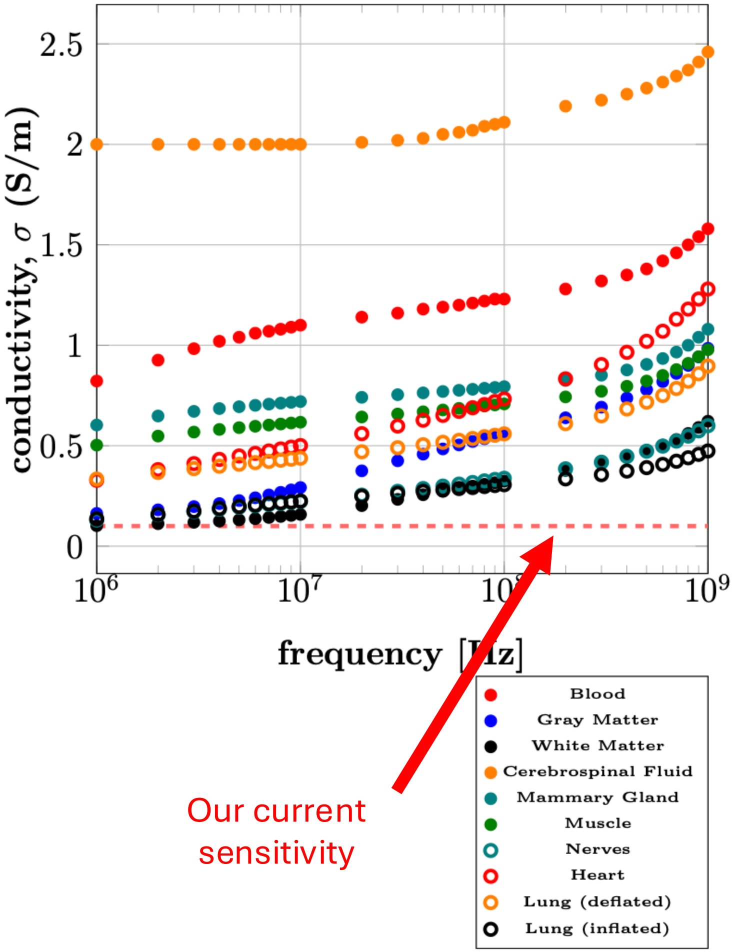




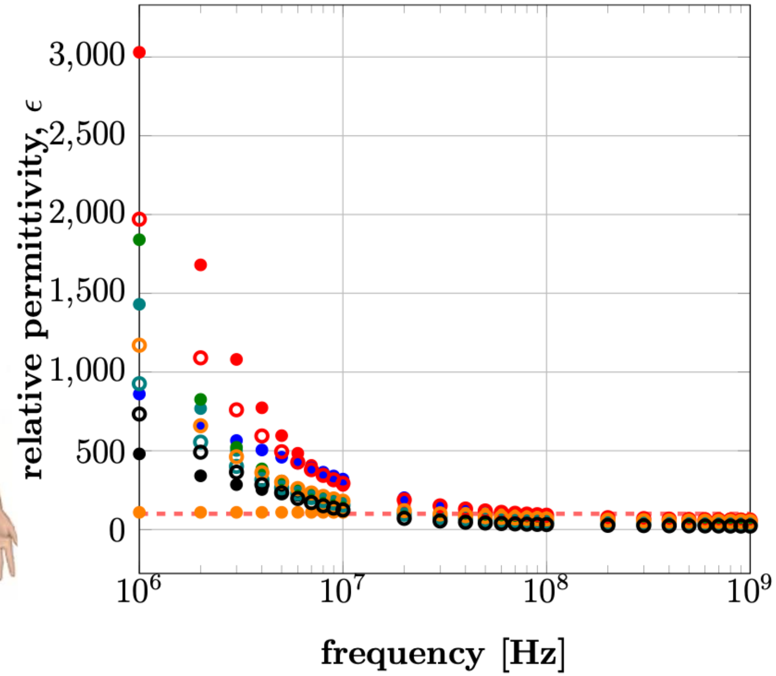
Goals:
Detection of brain, heart anomalies
by noninvasive conductivity
measurements

Zheng et al. **PRL** (2023).
Highlighted by Physics World and by APS
Physics Magazine

Quantum Magnetic Induction Tomography for tissue diagnostics



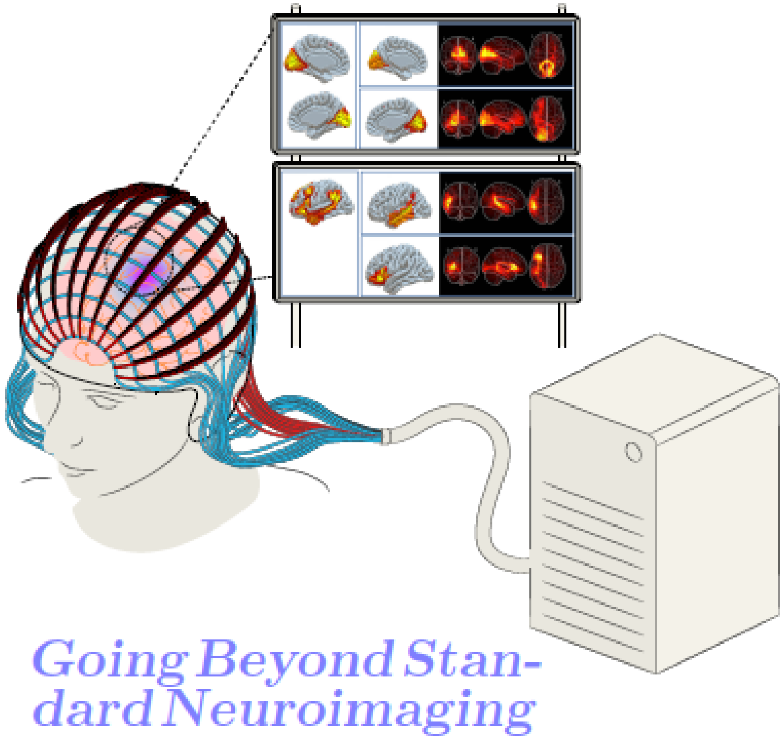
Our current sensitivity



Current Medical Diagnostic Technology such as MRI measure only water:

We aim to offer tissue specificity

Tomography by brain conductivity



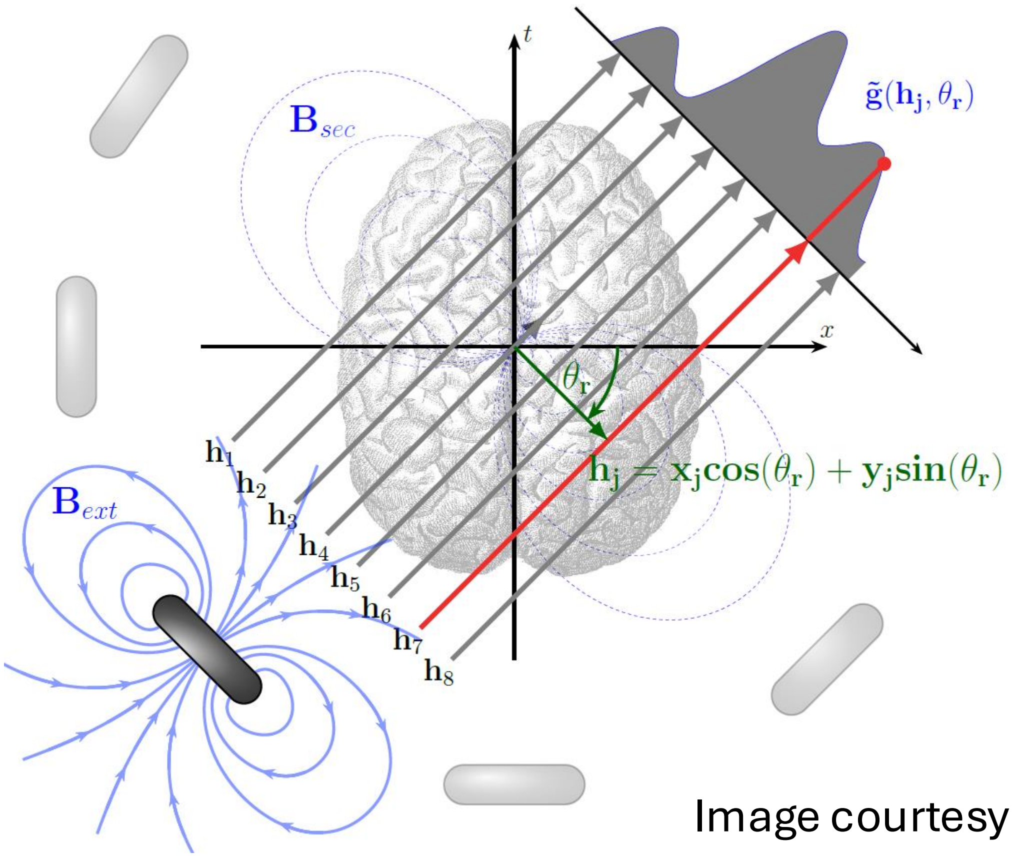


Image courtesy of D. Naik